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CRACK PREVENTION FOR MASSIVE CONCRETE MEMBERS AT EARLY AGE

Manuscript written by

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Figures, Photos on practical applications, test results measurements etc. will be presented during course.

1. PRACTICAL PROBLEM AND AIM OF COURSE

"Forecasting is difficult, especially of the future", Mark Twain

It is a fact that in massive concrete members cracks at the early age of concrete frequently arise. The main cause of such cracks is the evolution of the heat of hydration of cement. This process may lead to excessive heating-up of concrete and consequently to thermal stresses if the member is internally and/or externally restrained. Fig. 1 shows the problem for a wall cast on an old foundation. Which are the consequences of cracks? Water-tightness, durability (steel corrosion) of structure etc. may be impaired. Economic consequences can be very great.

It is also fact that many structural engineers in practice are not adequately trained to deal with transient phenomena such as early age temperature problems, restraint, stresses and the avoidance of cracks. It is the aim of this course to teach the basics of these phenomena.

2. EMPIRICAL MEASURES FOR CRACK PREVENTION AND NEW APPROACHES

In view of the consequences of cracking, engineers in practice and researchers have developed numereous preventive measures. Most of these measures are based on experience (viz. ACI Manual of Concrete Practice, Parts 1 and 3, latest edition). Fig. 2 presents an overview on these measures which relate to various fields of activity.

Mostly these measures are not accompanied by the calculation of temperatures and stresses in the structures. Hence, often decisions are made on good feeling, experience, intelligent quessing. Consequently, disappointment is frequent.

Nowadays, new approaches are being developed in Europe, Japan and also in the US (viz. ACI Materials Journal of the last years). These new approaches combine the experience - based measures of Fig. 2 with computer-aided engineering models to a holisitc decision tool. Fig. 3 shows the elements of such approach, which will follow along in this course.

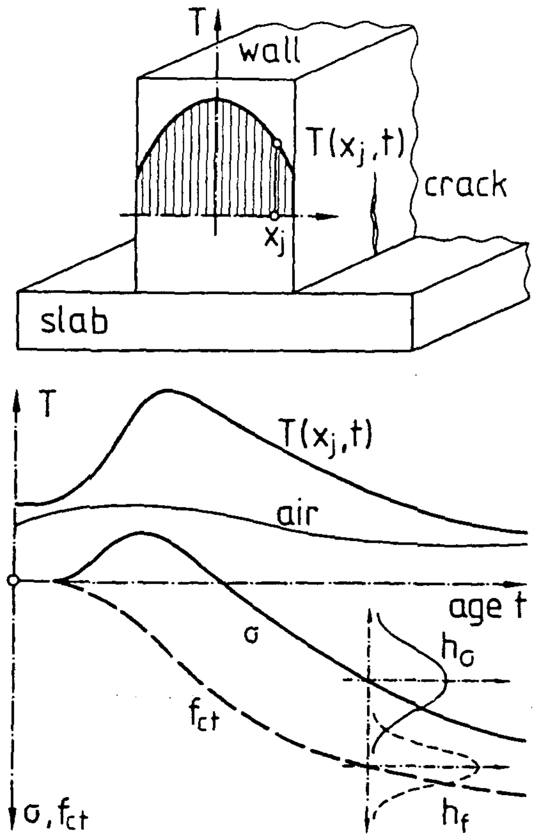


Fig. 1: Temperature, strength and stress

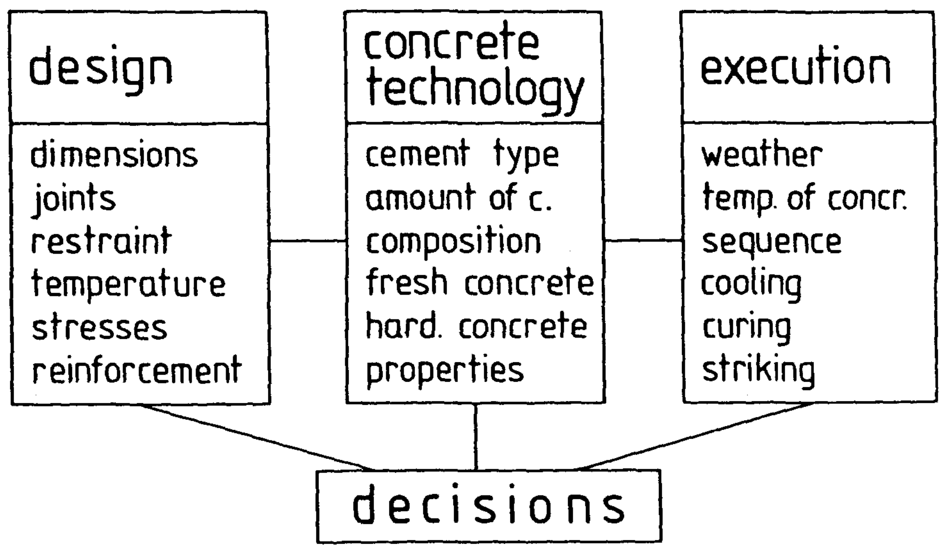


Fig. 2: Empirical ways of crack prevention

3. ENGINEERING MODEL FOR CRACK PREVENTION

Crack prevention usually encompasses a series of measures such as

- type and amount of cement and other hydraulic additives such as fly ash, all of these preferably with a low heat release
- low fresh concrete temperatur T_{c0}
- artificial cooling of concrete components e.g. in hot summer
- internal or external cooling of placed and hardening concrete
- choice of block lengths, expansion joints
- sequence of construction

an many others. Such measures must be planned and cause costs; hence they should not be futile. Crack repair is very costly.

Engineers deal with such measures in basically two situations:

- in the pre-design and tender phase: The engineer knows little to nothing about the concrete, weather etc. Uncertainty is high.

- in the execution phase: Contract has been awarded. Knowledge on concrete, construction sequence etc. grows gradually. Insecurity turns to certainty. But: the prize is fixed, and time is pressing.

Now to goals of the depicted engineering model: It should enable the engineer to make responsible decisions and to give reasonable advise to engineers on-site. As Fig. 3 shows, the model consists of several submodels, which will be outlined during the course. The model encompasses:

- knowledge, experience, data bases on various items, such as concrete composition from previous works, etc.

- test results on properties of concrete to verify and to adapt the material models of hydration, of thermal, hygral and mechanical properties of young concrete.

- preview of weather situation.

- model of restraint conditions.

- computer models for the calculation of fields of temperature, degree of hydration, stresses etc. and of
- critical zones prone to cracking.

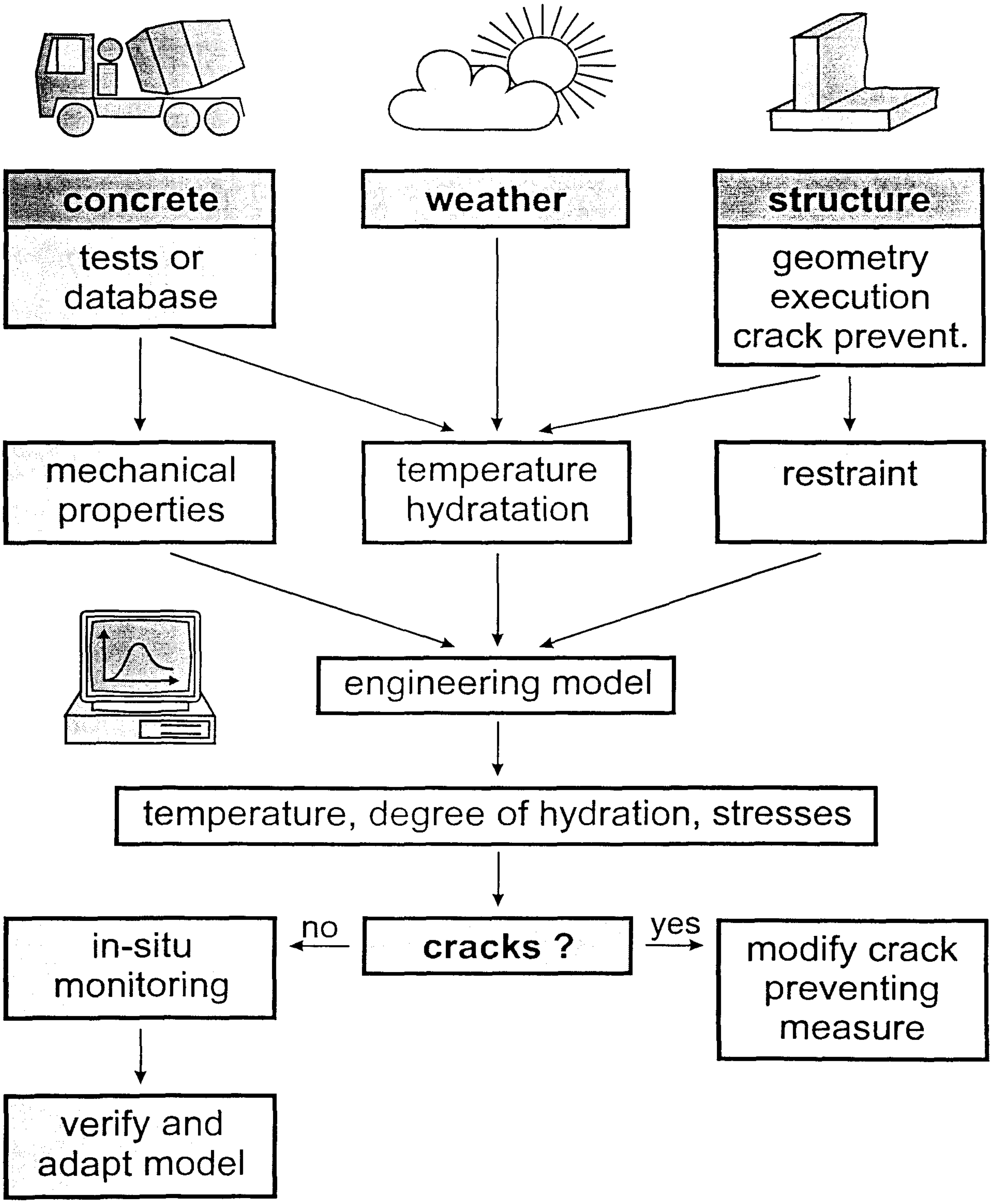


Fig. 3: Adaptive engineering model for crack prevention

4. ON HARDENING, LIBERATION OF HEAT OF HYDRATION AND DEGREE OF HYDRATION OF CONCRETE

4.1 Object

In the center of any measure to prevent early age cracks are the caloric, thermal, hygral and mechanical phenomena in course of hydration of concrete. In order to understand the rapid evolution of temperature generation and of concrete properties in a structural member, we have to deal with hydration.

4.2 Reactions and Formation of Microstructure

After the mixing of cement plus other hydraulic additives (e.g. ground blast furnace slag GBFS, fly ash PFA) with water, the hardening reactions set on. These reactions are summarized as hydration. Hydration is an exothermal process, during which the heat of hydration is liberated and the microstructure of cement stone is formed. This microstructure consists of the calciumsilicatehydrate phases CSH etc. In Fig. 4 the formation of the structure of hardened cement stone is schematically depicted.

The rate of reaction depends on several parameters such as:

- chemical composition of cement and hydraulic additives
- mineralogical phase composition of reactive phases (e.g. C_3S, C_2S, C_3A, C_4AF, glassy FA, GBFS etc.)
- granulometry of binder components
- water-binder-ratio
- regime of temperature during hardening
- additives (e.g. super plasticizers)

4.3 Degree of Hydration

The degree of hydration is the governing state parameter of hardening. It incorporates the real age of concrete and the temperature regime during hardening as well. It is defined by:

$$\alpha(T(t)) = \frac{m_{CSH}\,(T(t))}{m_{b0}} \quad [-] \tag{1}$$

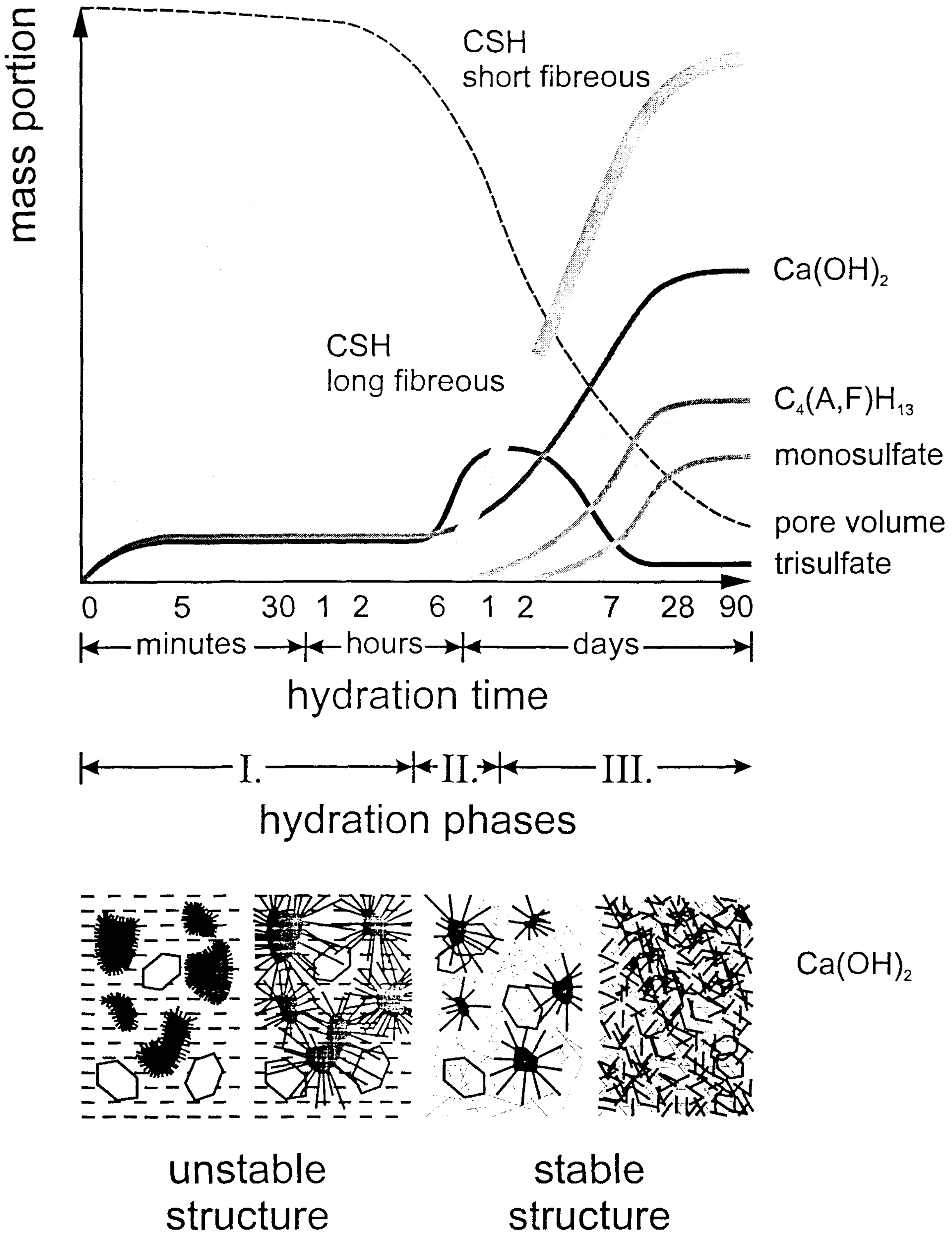


Fig. 4: Hydration phases and microstructure during hardening of cement stone

thereby:

$$0 \leq \alpha(T(t)) \leq 1 \tag{2}$$

With: m_{b0}, initial mass of total binder; m_{CSH}, part of total binder transformed into CSH + $Ca(OH)_2$ Portlandite.

The degree of hydration acc. Eq.(3) cannot be measured exactly up to now. An alternative way of the determination of α is its description via the liberated heat of hydration which can be measured. This definition pre-supposes that the formed mass of m_{CSH} is proportional to the liberated heat of hydration:

$$\alpha(T(t)) = \frac{Q(T(t))}{\max Q} \quad [-] \tag{3}$$

with: max Q, total heat release of concrete in kJ/m^3; Q(T(t)), heat release until age t along temperature path T(t), in kJ/m^3 concrete (max Q = total heat release of cement plus hydraulic binders). All mechanical properties can be expressed via the degree of hydration. Explaining figures will follow. (Conversion: 1 kJ = 0.278 Wh).

4.4 Determination of Heat Release

As Eq.(3) shows, two constituents must be determined. The denominator max Q cannot be readily measured, it must be assessed from the composition of cement and hydraulic binders. The nominator can either be estimated or directly measured.

- Measurement of Q(T(t))

The most appropriate method to measure the heat release is the adiabatic calorimetry. Fig. 5 shows an adiabatic calorimeter. A concrete probe of 10 to 30 dm^3 ($\triangleq$ litres) is placed in an heat-insulating box. By a temperature regulating system, the heat flux from inside to outside is totally obviated: $\partial Q/\partial r = 0$.

In course of hydration, the temperature increases steadily. In Fig. 6 test results are presented. The increase $\Delta T_{ad}(t)$ above different fresh concrete temperatures T_{c0} vs. the age t is plotted. The total temperature is then:

$$T_{ad}(t) = T_{c0} + \Delta T_{ad}(t) \tag{4}$$

which can be measured. Because of the relation

$$Q_{ad}(T(t)) = c_c\, \rho_c\, \Delta T_{ad}(t) \tag{5}$$

the heat release can be determined. With: c_c, specific heat of concrete [kJ/kgK]; ρ_c , density of fresh concrete [kg/m^3]; t, real age of concrete, counted from end of mixing in [h]. The definitions and ways to calculate c_c and T_{c0} are given in Appendix A.

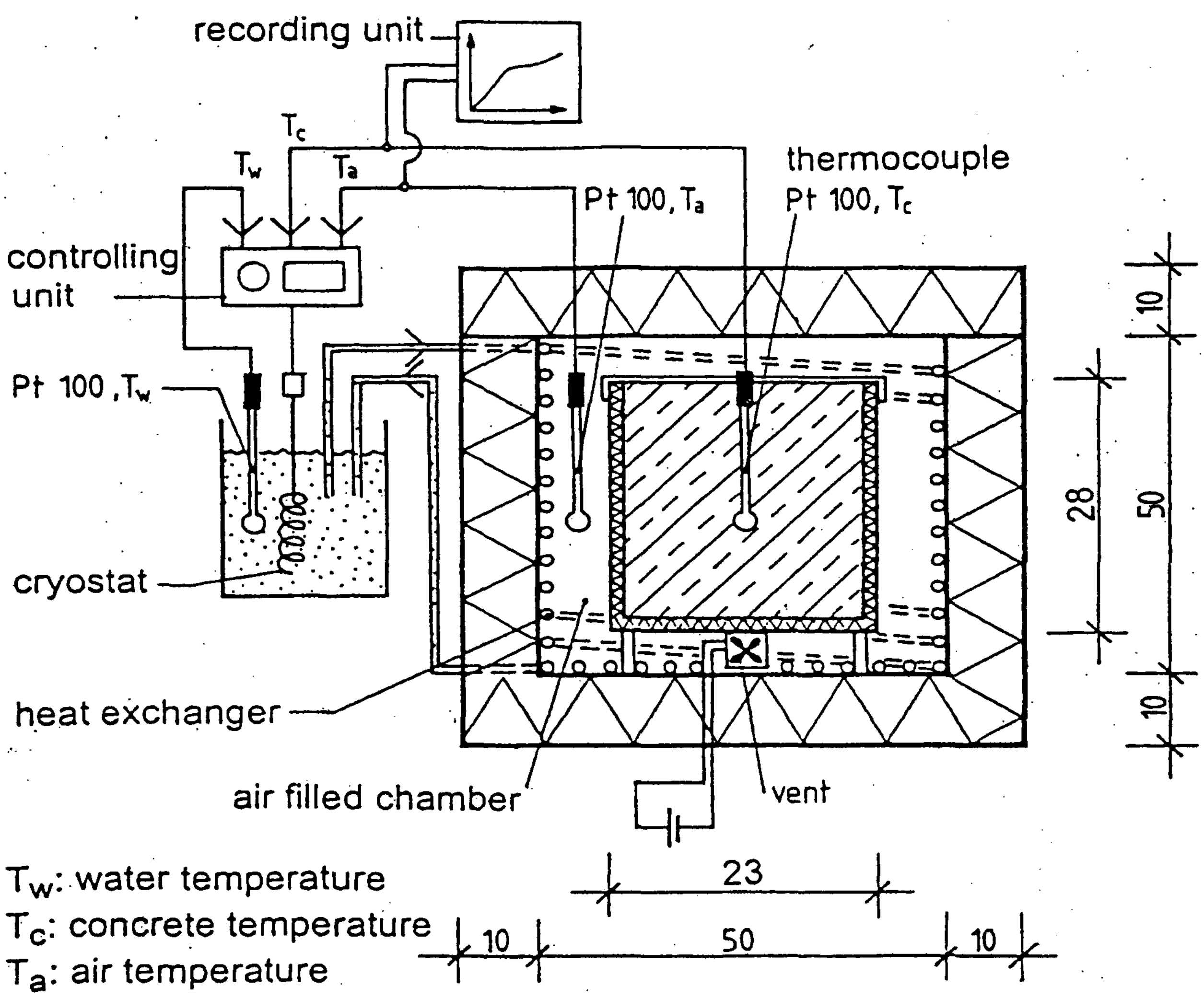


Fig. 5: Adiabatic Calorimeter

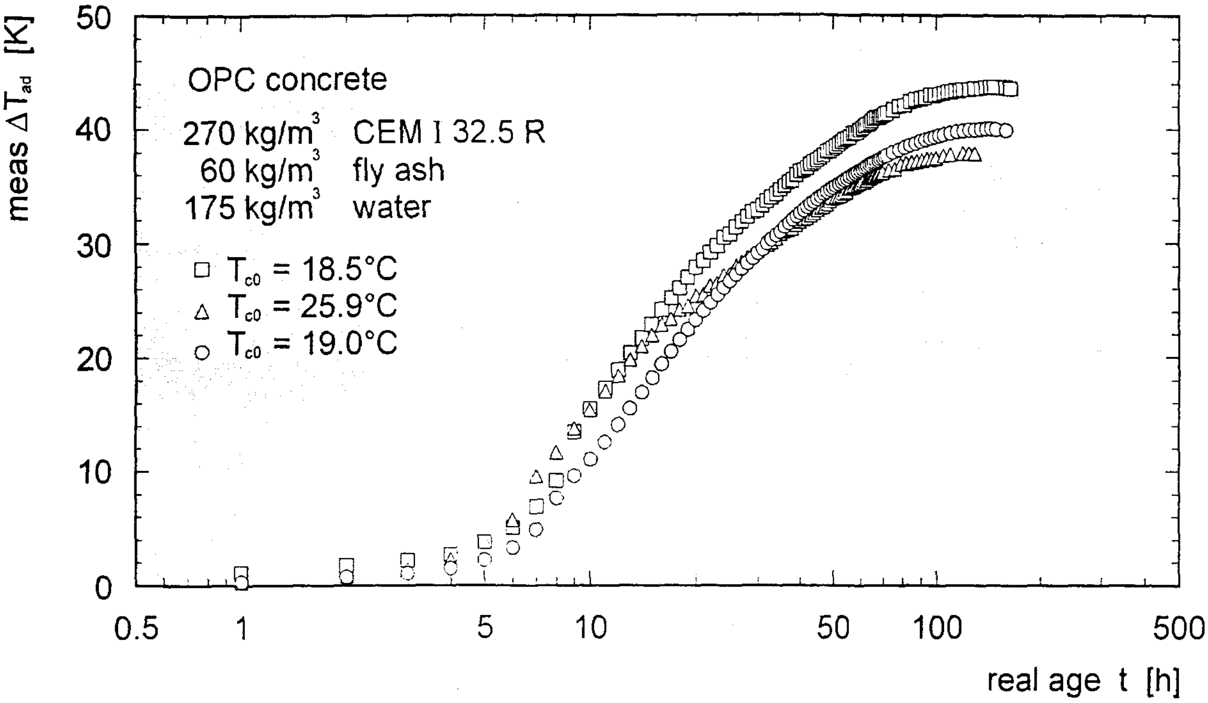


Fig. 6: Measured adiabatic temperature rise dependent on age and temperature of fresh concrete

After some time, temperature levels off. Hydration comes to its end. Hence, we could estimate max Q with:

$$\max Q = c_c \, \rho_c \max \Delta T_{ad}(t \rightarrow \infty) \qquad (6)$$

But as hydration still proceeds, though with slow pace, such estimation is inaccurate. Hence, we must look for a more appropriate way.

- Assessment of maximum heat release max Q

For the assessment of max Q we must know the content of cement C and of the hydraulic binders (kg/m^3 concrete) **and** the maximum heat release of cement and binders, per unit weight (kJ/kg).

Heat release of cement is a specific material property. The ASTM standards give values. Also from cement producers information may be obtained. Any cement contains a certain mass portion of portland cement clinker (e.g. 100 % clinker in ordinary PC; a GBFS-C may consist of e.g. 60 % clinker and 40 % GBFS).

The clinker consists of the reactive clinker minerals (which will form CSH- and CAH-phases in cement stone):

C_2S	Dicalciumsilicate
C_3S	Tricalciumsilicate
C_3A	Tricalciumaluminate
C_4AF	Tetracalciumaluminateferrite etc.

If we would know the mass portions of these minerals in the clinker and their intrinsic heat releases, we could calculate max Q_C for an ordinary PC by:

$$\max \hat{Q}_C = \sum (m_{C_2S} \cdot Q_{C_2S} + \ldots) \ [\text{kJ/kg of C}] \tag{7}$$

The procedure is shown in Appendix A, Eq.(4.1) for max ΔT_{ad} which is tied to max Q and max $\hat{Q}_C$ via Eq.(6). Certainly, also fly ash (FA) and slag (GBFS) contribute to heat release, viz. App. A.

The Eq.(6) and (7) are only valid for portland cements. If the cement contains GBFS slag and/or PFA fly ash, the total binder encompasses the PC-clinker and the hydraulic additives. This is also true if a hydraulic additive is incorporated seperately into concrete mixer. In either case App. A gives the necessary help to calculate the two theoretical values max Q and max T_{ad}.

- Influence of clinker minerals and of GBFC on the development of the degree of hydration

The clinker minerals C_2S, C_3S, ... etc. contribute to the resultant heat of hydration of cement quite differently. This fact is shown in Fig. 7. It is also true for the compressive strength.

4.5 Age and Temperature Dependence of Degree of Hydration

Hydration is a chemical process. Processes of that type are significantly temperature-dependent: low temperatures retard, high temperatures accelerate. This fact was already expressed by Eq.(3):

$$\alpha(T(t)) = \frac{Q(T(t))}{\max Q} \tag{3}$$

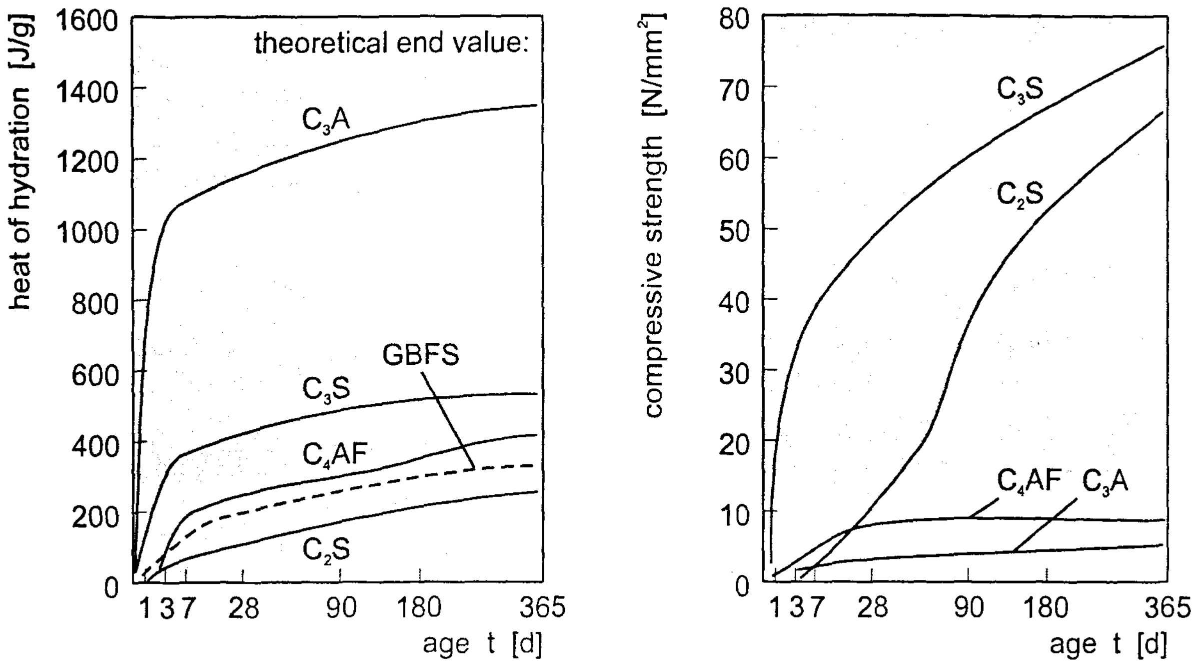


Fig. 7: Heat of hydration and compressive strength of clinker minerals and GBFS dependent on age at 20 °C

The value max Q is an intrinsic property of the total binder; it does not depend on age and temperature. It is a potential. The value Q(T(t)) shows, how this potential is exploited in course of time and temperature history T(t).

Let us begin with the three different temperature histories of Fig. 8: adiabatic, realistic in structure and isothermal at 20 °C. Because the total heat release depends on the age and temperature, these three histories will result in different functions Q(T(t)), center part of Fig. 8. But: they are all striving towards max Q.

If we draw a horizontal line at a specific heat release Q*, we see that this heat is reached at different ages $t_{ad} < t_2 < t_1$. What is true for Q*, must also be true for the degree of hydration because of:

$$\alpha^* = \frac{Q^*}{\max Q}$$

We have a lot of data and experience regarding hardening of concrete at 20 °C. If we cure concrete at elevated temperature T(t) > 20 °C, a certain strength f* is reached at shorter time, than when curing at 20 °C. Or we could mirror the problem: If we would know the time to reach the strength f* for curing at a temperature of 20 °C, how long would it take for T(t) > 20 °C?

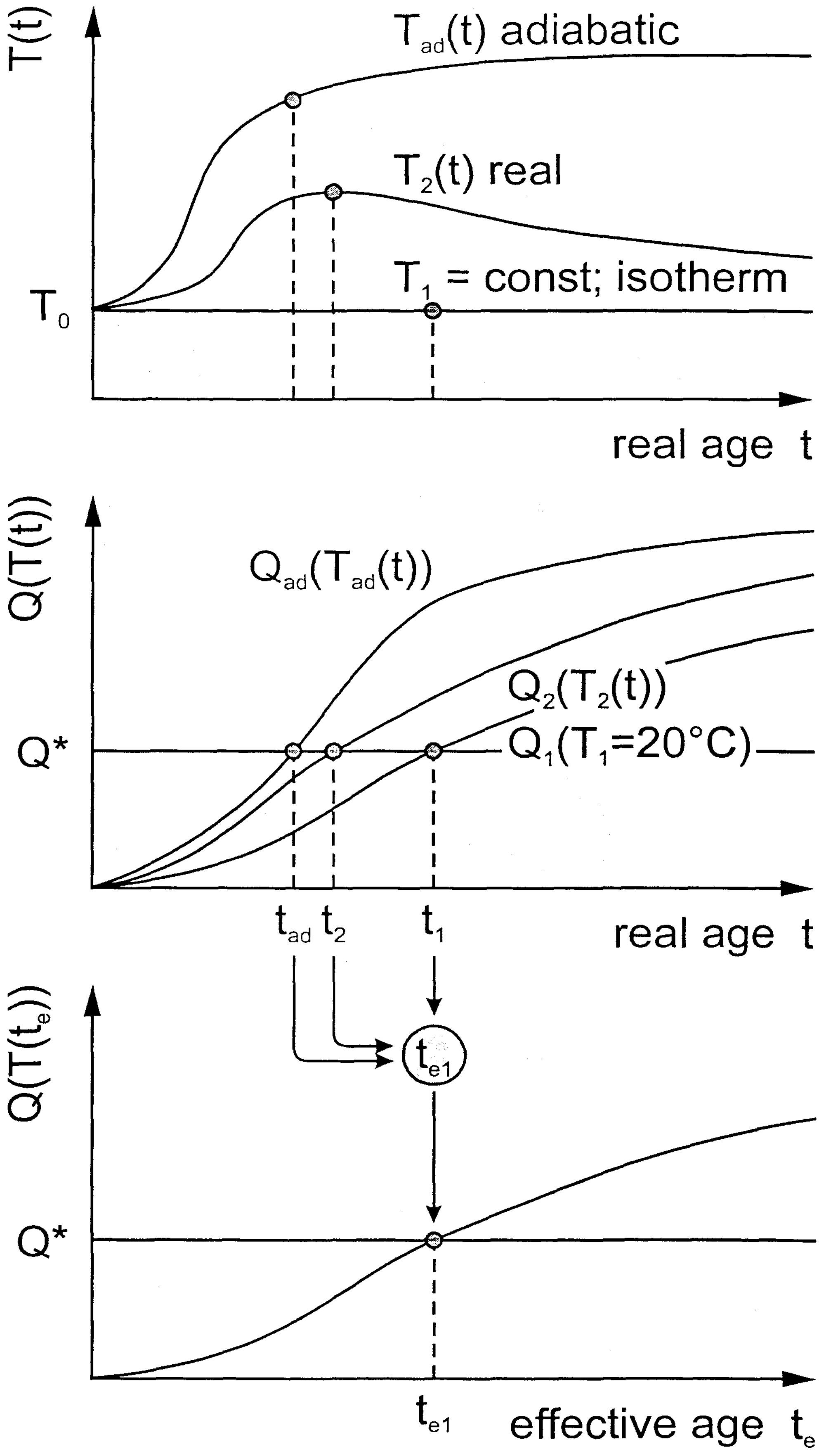


Fig. 8: Heat liberation of concrete dependent on temperature history; transformation of real age into effective age

This is for practice quite important (stripping time etc). The problem can be solved by an age-temperature-transformation. We use for that the Arrhenius law of rate kinetics of chemical processes. The rate of hydration can be expressed by

$$q(T(t)) = \frac{dQ}{dt} = p \cdot k(T(t)) \tag{8}$$

with: p, process factor (independent on temperature, material specific); k(T(t)), temperature dependent rate factor. The latter is expressed by

$$k(T(t)) \sim \exp - \frac{E_A}{R\, T_K(t)} \tag{9}$$

with:

E_A, activation energy [J/mol], needed to start hydration

R, universal gas constant, R = 8.315 [J/mol K]

$T_K(t)$, process temperature in Kelvin degrees, 1 K = 273 + T [°C]

The activation energy is expressed by:

$$\begin{aligned} E_A &= 33.5\,[\mathrm{kJ/mol}], \text{ if } T(t) \geq 20\,°\mathrm{C} \text{ or } T_K(t) \geq 293\,\mathrm{K} \\ E_A &= 33.5 + 1 \cdot 47\,(20\,°\mathrm{C} - T\,°\mathrm{C})\,[\mathrm{kJ/mol}], \text{ if } T(t) < 20\,°\mathrm{C} \end{aligned} \tag{10}$$

Let us now deal with the histories $T_2(t)$ and T_1 = 20 °C = const. of Fig. 8. We can express the heat release Q* by the rate of heat release:

$$q = \int_0^t \frac{dQ}{dt} \cdot dt \tag{11}$$

$$Q_2^*(T_2(t)) = p_2 \int_0^{t_2} \exp - \frac{E_A}{R \cdot T_{2K}(t')}\, dt' \tag{12}$$

and

$$Q_1^*(T = 20\,°\mathrm{C}) = p_1 \int_0^{t_1} \exp - \frac{E_A}{R \cdot 293}\, dt' = p_1 \cdot \exp\left(-\frac{E_A}{R \cdot 293}\right) \cdot t_1 \tag{13}$$

Because of $p_1 = p_2$ and $Q_2^* = Q_1^* = Q^*$, we arrive at

$$t_1 = t_e = \int_0^{t_2} \exp \frac{E_A}{R} \left[\frac{1}{293} - \frac{1}{T_{2K}(t')} \right] dt' \tag{14a}$$

By the age t_1, called effective age t_e, the equivalent duration of hardening at 20 °C - needed to attain the same hydration as process $T_2(t)$ until t_2 - is expressed. The effective age is also called equivalent age or maturity. Eq.(14) can also be written as sum:

$$t_e = \sum_{i=0}^{n} \exp \frac{E_A}{R} \left[\frac{1}{293} - \frac{1}{\widetilde{T}_{iK}} \right] \Delta t_i \tag{14b}$$

with: Δt_i, time interval; $\widetilde{T}_{iK} = (T_{iK} + T_{i+1K})/2$, in the center of interval Δt_i in [K].

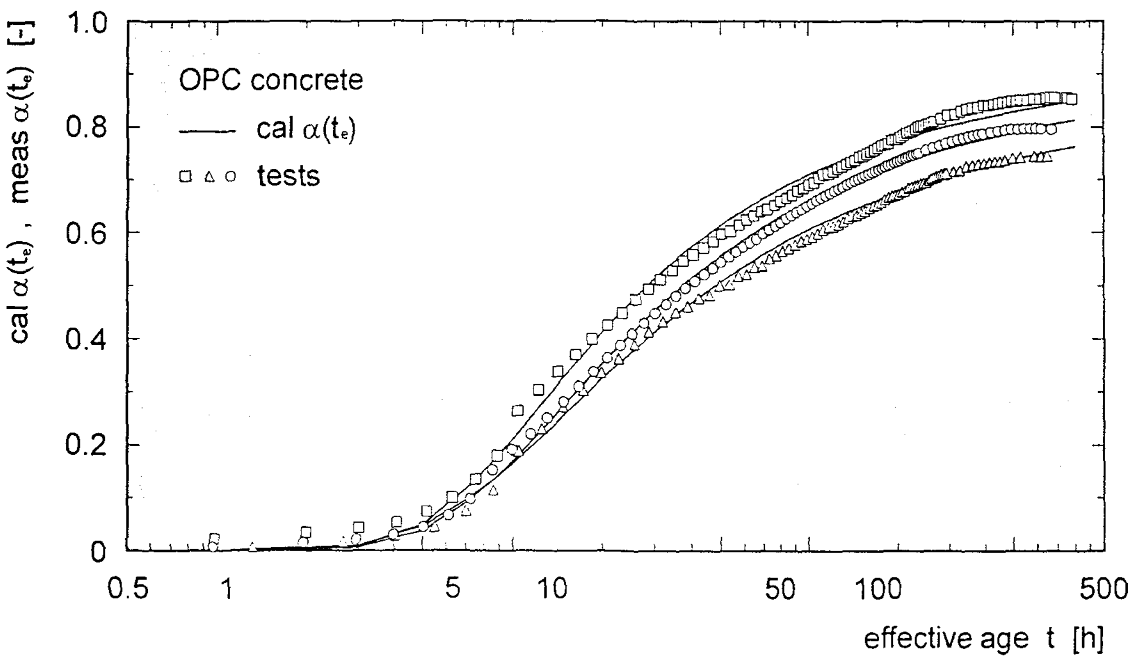


Fig. 9: Degree of hydration dependent on effective age and temperature of fresh concrete - tests and model

Fig. 8, bottom part, shows that by transforming all histories with the effective age, we obtain one unique release function $Q(t_e)$. Because the ordinate corresponds to

$$Q(T(t)) = \alpha(T(t) \max Q, \tag{15}$$

and

$$Q(t_e) = \alpha(t_e) \max Q, \tag{16}$$

this lower curve essentially depicts the degree of hydration. In Fig. 9 test results for variable fresh concrete temperatures are shown.

4.6 Model of Degree of Hydration

4.6.1 Jonasson's model

For the computation of temperatures in the structure, we need a mathematical model of the degree of hydration:

$$\text{cal}\,\alpha_j(t_e) = \frac{Q_j(t_e)}{\max Q} \qquad [-] \tag{17}$$

With j, space coordinates x,y,z are meant; α is variant in space (location) and effective age. For max Q viz. App. A and section 4.4.

The most appropriate model is Jonasson'model:

$$\alpha_j(t_e) = \exp - \left[\ln\left(1 + \frac{t_e}{t_k}\right)\right]^{c_1} \tag{18}$$

If α is plotted vs. $\log_n(t_e)$, we obtain a S-type curve, Fig. 9. This curve starts with a horizontal tangent (dormant phase of hardening) and strives-after a steep rise-towards - $\alpha \rightarrow 1$ for $t_e \rightarrow \infty$, end of hydration.

The parameters t_k and c_1 are specific for the cement and its amount, for the additional hydraulic binders and their amounts, for the fresh concrete temperature etc.

There exist also other models, outlined in App. A.

4.6.2 Determination of model parameters

There are several ways of determination of the model parameters.

- **On basis of adiabatic heat release tests**

Because

$$\text{meas}\,\alpha(t) = \frac{\text{meas}\,\Delta T_{ad}(t)}{\text{cal}\,\max \Delta T_{ad}} \tag{19}$$

can be derived from the adiabatic test, the determination of the parameters t_k and c_1 of Eq.(18) can be performed by non-linear regression. This is also true if the Danish or the Shrinkage Core Model (App. A) are applied.

For certain concrete compositions and cements, the data in App. A give advise regarding the suitable choice of the parameters t_k and c_1 for the evaluation of $\alpha(t_e)$ acc. to Eq.(18).

- **On basis of normative rules, literature etc.**

The Japanese model, shown in App. A, presents relations for specific types of cement and for the amount of cement C such as:

$$\text{cal max } \Delta T_{ad} \approx A \cdot C + B \tag{20}$$

With A and B dependent on fresh concrete temperature and type of cement.

The degree of hydration is given there for the hydration process in the adiabatic calorimeter by:

$$\alpha_{ad}(t) = 1 - \exp(-\gamma t) \tag{21}$$

with: t, real age; $\gamma = a \cdot C + b$, a and b being dependent on the type of cement and the fresh concrete temperature (viz. App. A). With Eq.(14) the "adiabatic" degree of hydration can be transformed into $\alpha(t_e)$, as has been shown in sec. 4.6.

Relations of this kind are based on tests, experience etc. Therefore, they will contain uncertainties. They are, hence, primarily suitable for the rough estimation of temperatures and stresses in a structure (e.g. in the pre-planning and bidding phase).

5. PROPERTIES OF YOUNG CONCRETE

5.1 Scope

For the assessment of stresses and eventual cracks in a young concrete structure the following properties of concrete must be known or suitably assumed

- axial tensile strength f_{ct}
- compressive cylinder strength f_c

- Young's moduli in tension E_{ct} and in compression E_c
- stress-strain lines σ-ε for tension and compression
- creep and relaxation functions especially for axial tension
- thermal expansion and autogeneous shrinkage

For these properties material models are necessary which can be applied in the pre-planning/ tendering phase and also in the execution phase. For the reliable forecast of hydration, temperatures and stresses, the comprehensive testing of the prospective concrete of the on-coming work is indispensable. Such tests can be performed in the execution phase of structure. However, in the pre-planning phase etc., little is known, not only about the concrete etc. Nevertheless, also for this situation, we must have tools of reasonable accuracy. In this course, we shall concentrate on engineering models for the pre-planning phase.

The evolution of the mechanical properties at early age is characterized by rapid change. Fig. 10 shows that this evolution does not start immediately after placing and compaction of fresh concrete. In a first "dormant phase" the concrete may be regarded as fluid. Hence, no stresses will be generated in the dormant phase, although hydration has already started. The dormant phase may last - depending on many parameters - for 3 to 8 hours after mixing. From then on, concrete may be regarded as solid matter. There certainly are fluent transitions. From a specific initial degree of hydration α_0 on, the mechanical properties will arise, stresses will be generated.

5.2 Compressive Strength

5.2.1 Definition and classification

The compressive strength of concrete is tested on e.g. cylinders h/d = 2 (300/150 mm). For the design and execution, the compressive strength will be specified. Furthermore, concrete quality is controlled. E.g. in the US a specific strength is for 28 d (characteristic strength):

$f_c' = 4000$ psi ($\hat{=}$ x 0.0069 = 27.6 MPa (N/mm^2))

Quality control must assure that the required average compressive strength at the age of 28 d must at least be:

$$f_{cr}' \geq f_c' + 1200 \text{ psi } (\hat{=} \; 35.8 \text{ MPa}) \qquad (22)$$

Internationally, the strength classes are defined by C .... E.g. in the EURONORM 1992, pt. 1 or in CEB-FIP Model Code 90:

$$\begin{aligned}&\text{C 30 } (\hat{=} \text{ characteristic cylinder strength at 28 d in MPa})\\ &f_{ck} = 30 \text{ MPa} \hat{=} f_c'\end{aligned} \tag{23}$$

The corresponding mean strength at 28 days is:

$$f_c = C + 8 \text{ MPa} \hat{=} f_{c28} \tag{24}$$

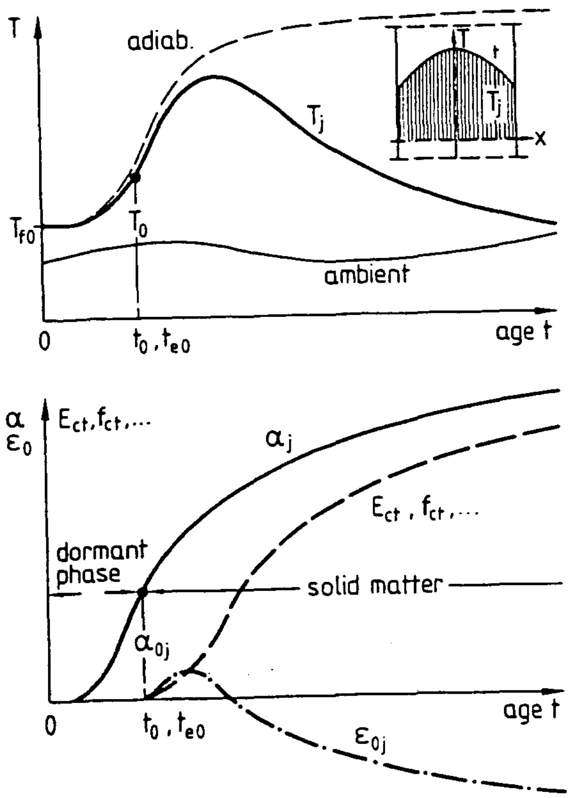


Fig. 10: Fluid to solid transition for young concrete

5.2.2 Compressive strength in the structure

For structural design, the engineer has to decide the necessary strength class C. For crack prevention, however he has to know the strength in the structure yet to be built. His starting point will nevertheless be the strength class C. Records of running concrete productions show that the mean and characteristic strength of concrete (tested on cylinders) are significantly exeeded. On the other hand, we have to take into account that the compaction of concrete in the structure is less intense than that in steel moulds. These counter-acting effects can be taken into consideration by stipulating for the mean effective compressive strength of the concrete in the structure at the age of 28 d (subscript e for effective):

$$f_{ce} \approx C + 8\ \text{MPa} \mathrel{\hat{=}} f_{ce28} \tag{25}$$

5.2.3 Material model of compressive strength

The compressive strength f_c can be expressed in terms of the degree of hydration:

$$f_c(\alpha) = f_{c1}\left(\frac{\alpha - \alpha_0}{1 - \alpha_0}\right)^n \tag{26}$$

with: f_{c1}, theoretical final value at $\alpha = 1$; α_0, degree of hydration at the end of dormant phase (Fig. 10); n, parameter. All these parameters can be determined by compressive strength tests at several values of effective age t_e, Eq.(14) and (18). The exponent n can be assumed: $n = 3/2$. Dependence of f_c on α is shown in Fig. 11.

5.2.4 Age and temperature dependence of compressive strength

For the purpose of crack prevention, the compressive strength of concrete does not play an eminent role. But for the computation of thermal stresses the stress-strain line dependent on effective age t_e and degree of hydration $\alpha(t_e)$ is needed. Hence, we have know more than the 28 days strength. Needed are $f_c(t_e)$ or $f_c(\alpha)$.

The dependence of compressive strength on age and temperature can be expressed with Eq.(27), with degree of hydration acc. to Eq.(18):

$$f_c(t_e) = f_{c1}\frac{1}{(1-\alpha_0)^{3/2}}\left\{\exp-\left[\ln\left(1+\frac{t_e}{t_k}\right)\right]^{c_1} - \alpha_0\right\}^{3/2} \tag{27}$$

The effective age is given by Eq.(14). Certainly, the parameters t_k and c_1 must be known from adiabatic tests or must by be assumed.

A simple way to express the dependence of f_c on t_e is proposed in the CEB-FIP Model Code 90:

$$f_c(t_e) = \beta_{cc}(t_e) \cdot f_{c28} \tag{28}$$

With: f_{c28}, mean compressive strength at the age of 28 d and vor curing at 20 °C, Eq.(24); $\beta_{cc}(t_e)$, age factor:

$$\beta_{cc}(t_e) = \exp\left\{ s\left[1-\left(\frac{28}{t_e}\right)^{1/2}\right]\right\} \quad [-] \qquad (29)$$

with: t_e, eff. age in days; s = 0.38 for GBFC and s = 0.25 for OPC, slowly hardening. We see that for t_e = 28 d, β_{cc} = 1.

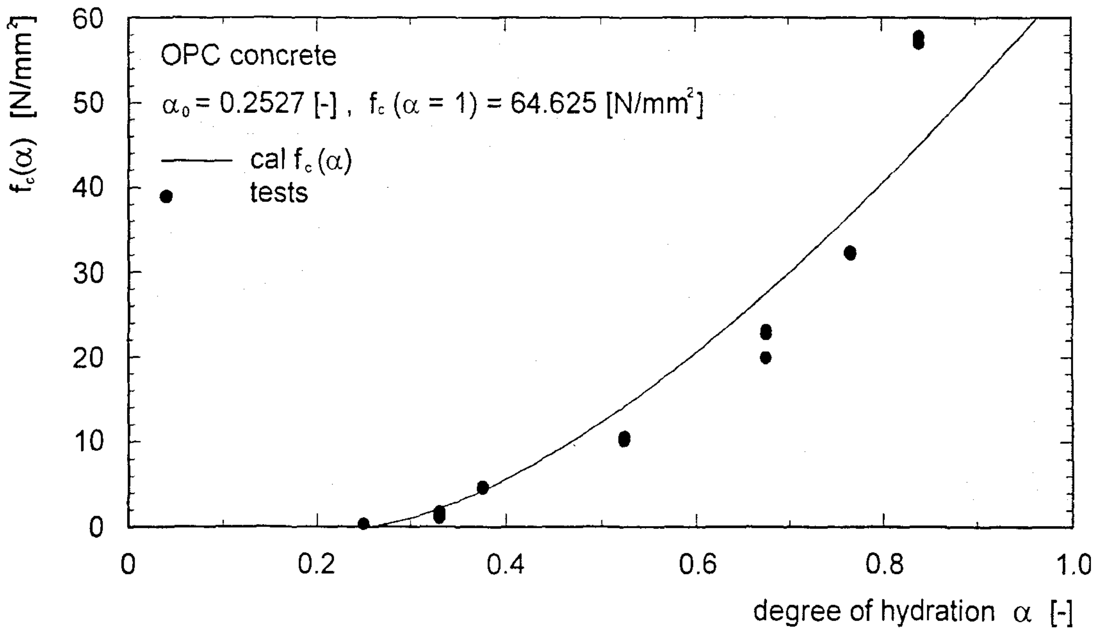


Fig. 11: Concrete cylinder strength dependent on degree of hydration - tests and model

5.3 Axial Tensile Strength

5.3.1 Relevance and material model

The tensile strength is **the** resistance against cracking. If a concrete specimen is cured at a defined temperature T(t) and is then tested at a specific real age, its effective age t_e and degree of hydration $\alpha(t_e)$ are known, provided heat release tests were performed aforehand. Fig. 12 and 13 show results of tensile tests for which these pre-conditions were met. Between the mean tensile strength and the degree of hydration α a linear relationship may be assumed. With that we arrive at the **material model**:

$$f_{ct} = f_{ct1}\frac{\alpha-\alpha_0}{1-\alpha_0} \qquad (30)$$

The theoretical end value f_{ct1} for $\alpha = 1$ and the initial value α_0 are found by regression of the test results. The initial value α_0 depends on the type and amount of cement and of hydraulic binders. For GBFS cement α_0 is between 0.30 and 0.35; for OPC with 25 % FA it is between 0.15 and 0.25 and for 100 % OPC, we obtained values between 0.10 (high-strength) and 0.2 (medium strength). The use of chemical additives such as super-plastizisers and retarders may prolong the dormant phase.

5.3.2 Age and temperature dependence of tensile strength

The strength development of a GBFC-concrete is plotted in the upper part of Fig. 13 and in the lower part for OPC-concrete. With the relationship already developed in sec. 4 for $\alpha(t_e)$; Eq.(18) and $t_e(T(t))$; Eq.(14), we can describe the dependence on t_e:

$$f_{ct}(t_e) = \frac{f_{ct1}}{1-\alpha_0}\left\{\exp - \left[\ln\left(1+\frac{t_e}{t_k}\right)\right]^{c_1} - \alpha_0\right\} \tag{31}$$

The parameters for these concretes are:

	GBFC-concrete	OPC-concrete
f_{ct1} [MPa]	3.23	2.90
c_1 [-]	-1.08	-1.06
t_k [h]	17.23	11.06
α_0 [-]	0.35	0.15 ÷ 0.25

There exist also other models to describe the development of tensile strength. In Fig. 13 the model of MC 90 was applied to describe the behaviour. It is quite suitable.

5.3.3 Tensile strength in the structure

In the execution phase, the tensile compressive strength of the actual concrete can be tested. However, we usually obtain thereby strength values which are higher than those of the concrete in the structure. The reasons for these differences are:

- better compaction of concrete in mould than in structural member
- thermal and hygral stresses in structure cause microcracks etc.

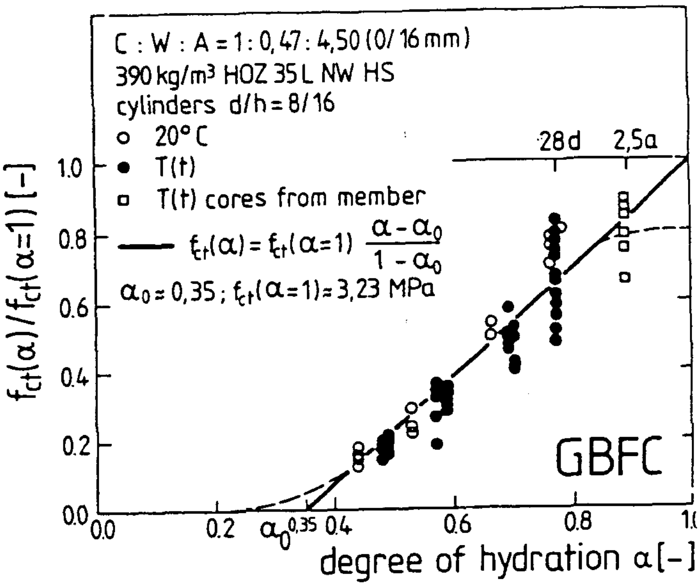


Fig. 12: Axial tensile strength vs. degree of hydration

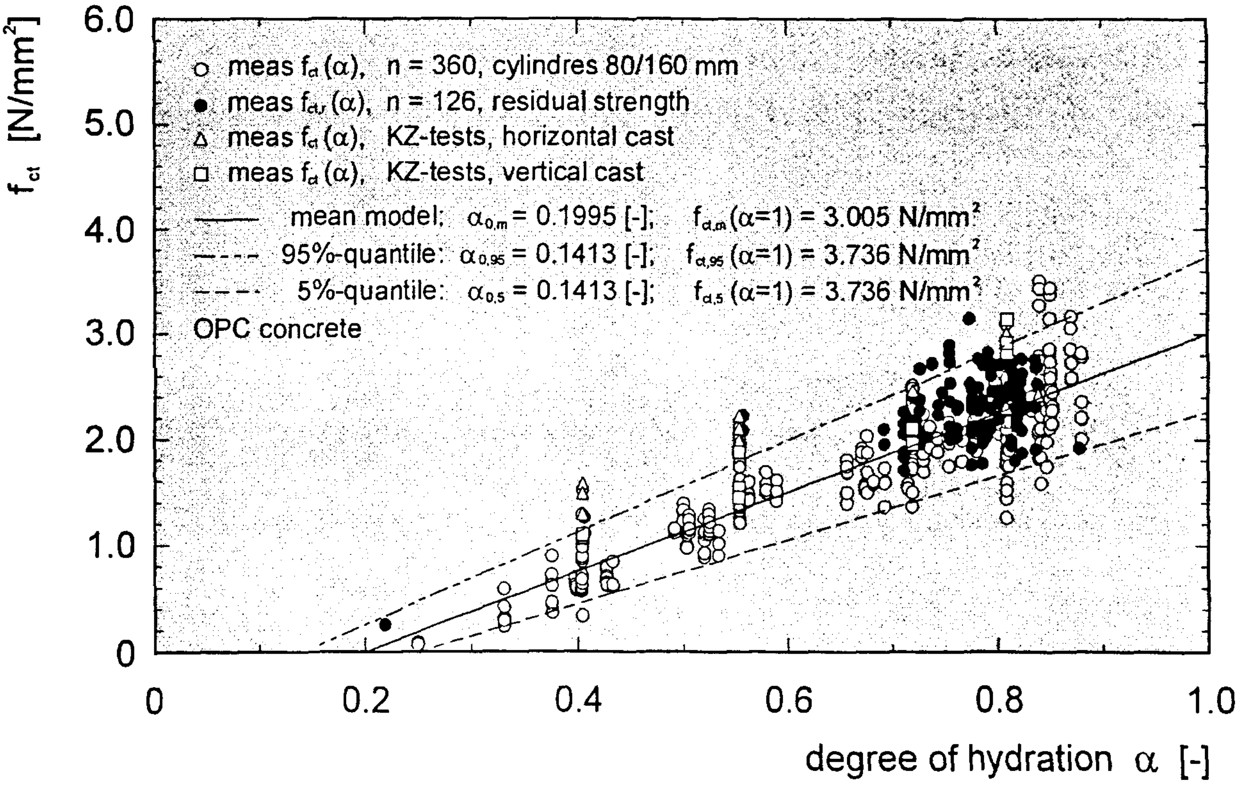


Fig. 13: Axial tensile strength dependent on degree of hydration - tests, model and scatter

There exist only approximate ways to take these effects into account. Many laboratory tests prove that the mean axial tensile and the mean compressive strength of concrete are linked as described by the empirical relationship (f_{ct}, f_c in MPa):

$$f_{ct}(t_e) = c\, f_c^{2/3}(t_e) \tag{32}$$

with: f_{ct}, mean tensile strength; f_c, mean compressive strength; c, factor established by tests. It was found by tests that the parameter c is independent on the effective age; it approximately amounts to c = 0.25.

The Eq.(30) and (32), being "laboratory models", must now be transferred into the structure. Tests prove that the mean effective tensile strength $f_{cte}(t_e)$ in the structure can be described by a reduction factor of ≈ 0.85 which covers the effects of micro-cracking due to long-term thermal tensile stresses:

$$f_{cte}\left(t_e\right) \approx 0.2\, f_{ce}^{2/3}(t_e) \tag{33}$$

with: $f_{ce}(t_e)$ mean effective compressive strength [MPa].

There exist several models to describe the strength development vs. effective age (viz. App). The already mentioned MC 90 model is rather simple. With Eq.(25), (28), (29) and (33), we obtain

$$f_{cte}\left(t_e\right) \approx 0.2\, \beta_{cc}^{2/3}(t_e)\, f_{ce,28} \qquad (33)$$

The lines in Fig. 14 were found with Eq.(33).

5.4 Moduli of Elasticity and Stress-Strain Lines

5.4.1 Components of deformations

Concrete subjected to axial tension exhibits several components of deformation. In short-term loading with a constant strain rate, we observe elastic and plastic deformations. The latter is caused by microcracks. Fig. 15 shows the stress-strain-line, which will dealt with later.

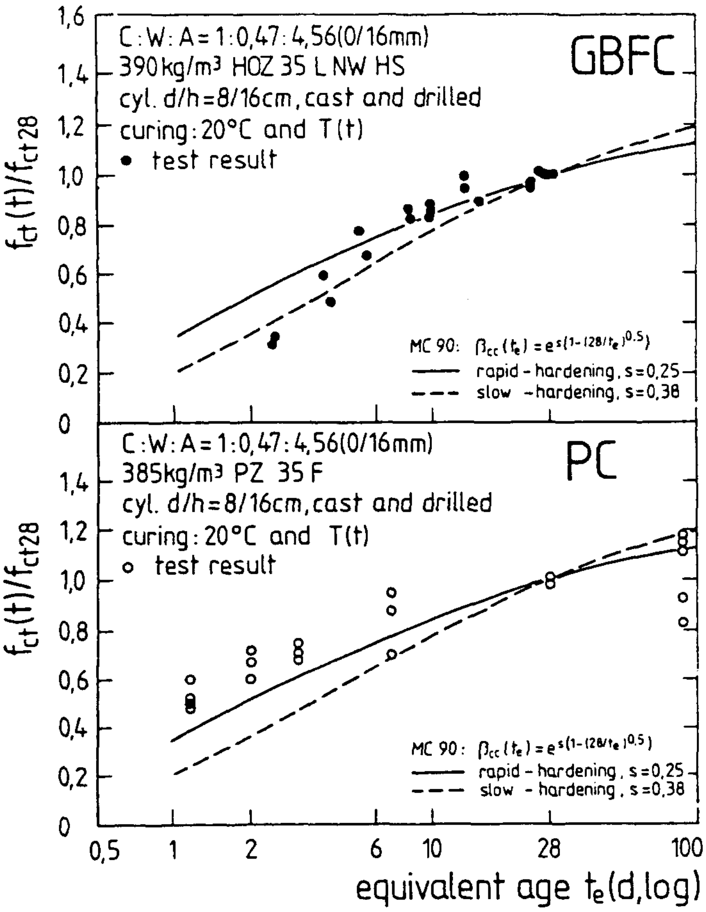


Fig. 14: Tensile strength vs. effective age

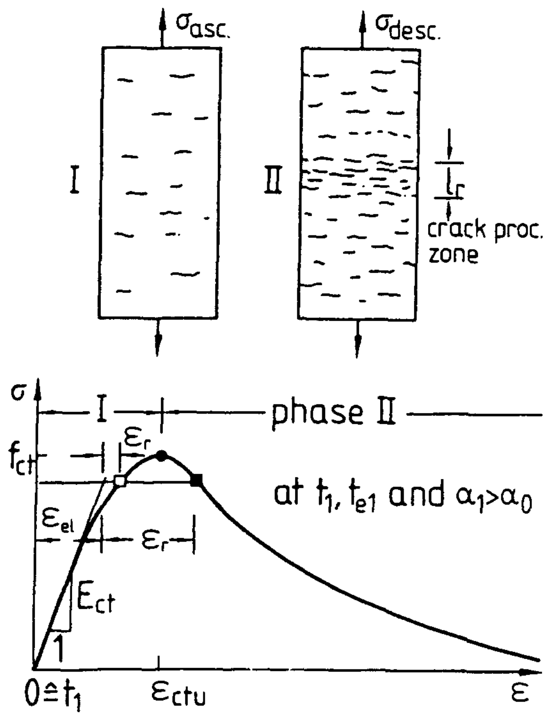


Fig. 15: Stress-strain line in tension

If we stress the specimen to a stress $\sigma < f_{ct}$ and then maintain this stress constant for some time, we observe an increase of strain dependent on time under stress. This response is defined as creep. If we stress the specimen to a certain strain being kept constant for some time, we observe a drop-off of the initial stress dependent on time. This responce is called stress relaxation. Both phenomena, creep and relaxation are two sides of one coin: visco-elasticity.

For stress computations, elasticity, plasticity and viscoelasticity must be known and modelled.

5.4.2 Young's moduli

The modulus of elasticity in **axial tension E_{ct}** ist defined as the slope of the σ-ε-line between $\sigma = 0$ and $\sigma = f_{ct}/2$, viz. Fig. 15. Our tests have shown that E_{ct} can be expressed in terms of degree of hydration, Fig. 16:

$$E_{ct} = E_{ct1}\left(\frac{\alpha - \alpha_0}{1 - \alpha_0}\right)^{1/2} \tag{34}$$

with: E_{ct1}, modulus at $\alpha = 1$; this value can be found by regression of test data. Tests have proved that Eq.(34) also represents the Young's modulus in the structure. Eq.(34) can also be used - in conjunction with Eq.(18), $\alpha(t_e)$ - to describe the age-dependence of $E_{ct}(t_e)$.

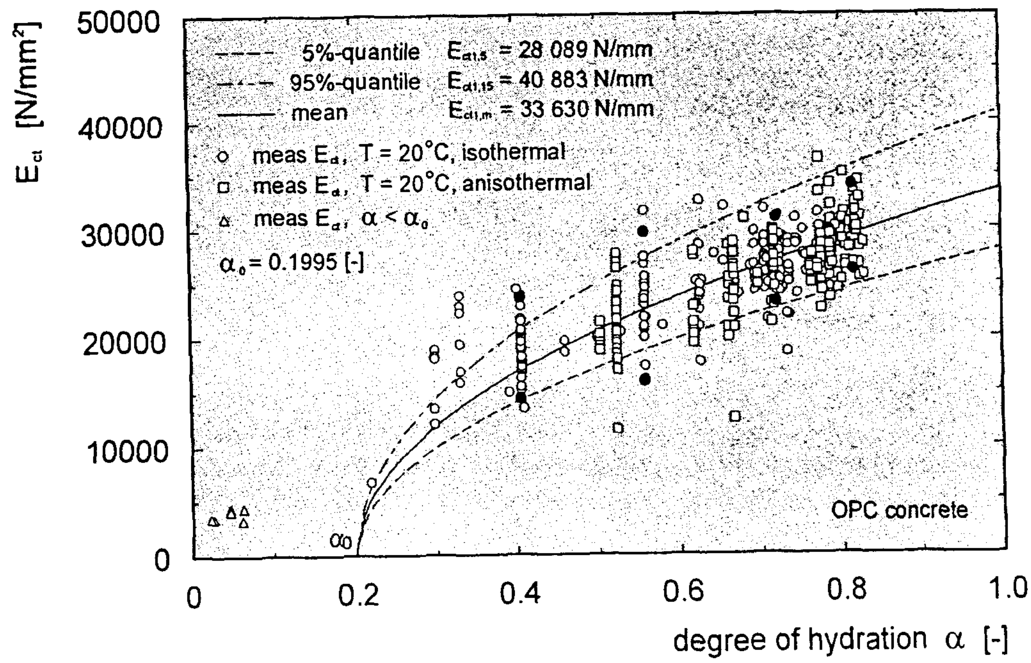


Fig. 16: Tensile modulus of elasticity dependent on degree of hydration - tests, model and scatter

In the pre-planning phase, tensile test data are usually not available. With Appendix A we estimate the parameters α_0, t_k and c_1. Then with Eq.(18), an approximate function $\tilde{\alpha}(t_e)$ can be derived. The end value E_{ct1} in Eq.(34) must also be estimated. This is usually done via the compressive strength. For a slowly hardening cement we obtain

$$\tilde{E}_{ct1} \approx 11300 \cdot f_{ce}^{1/3} \qquad [\mathrm{MPa}] \tag{35}$$

with: f_{ce} from Eq.(25). Example: C = 30 MPa; f_{ce} = 38 MPa; E_{ct1} = 36960 MPa. For a rapidly hardening cement, the first factor in Eq.(35) is 10800 (viz. CEB FIP-Model Code MC 90).

For approximative computations, we may assume that the Young's modulus in compression is identical with that in tension: $E_c \approx E_{ct} \hat{=} E$. Thus, we may write:

$$\tilde{E}(t_e) = \tilde{E}_{ct1}\left(\frac{\tilde{\alpha} - \alpha_0}{1 - \alpha_0}\right)^{1/2} \tag{36}$$

or by using Eq.(18) for the degree of hydration:

$$\tilde{E}(t_e) = \tilde{E}_{ct1}\sqrt{\frac{1}{1-\alpha_0}}\left[\exp - \ln\left(1 + \frac{t_e}{t_k}\right) - \alpha_0\right]^{1/2} \tag{37}$$

with: t_e, acc. to Eq.(14).

In MC 90 a simple approach is presented to describe the age-dependence of Young's modulus:

$$\tilde{E}(t_e) = E_0\,\beta_E(t_e) \tag{38}$$

with: E_0, modulus at 28 d; f_{ce}, viz. Eq.(25).

$$E_0 = 10000\,f_{ce}^{1/3} \tag{39}$$

$$\beta_E(t_e) = \beta_{cc}^{1/2}(t_e) \tag{40}$$

$$\beta_{cc}(t_e) = \exp s \cdot \left[1 - \left(\frac{28\,t_1}{t_e}\right)^{1/2}\right] \tag{41}$$

and with: t_e, effective age in days; t_1 = 1 d; s = 0.38 for slowly hardening cement; s = 0.25 for rapidly hardening cement.

5.4.3 Stress-strain lines

Let us return to Fig. 15. The stress-strain line in tension consists of two parts (phases). In the ascending branch, the deformation comprises both elastic strains ε_{eL} and micro-plastic strains ε_r. The latter increase as the tensile strength is approached; they are associated with diffusely distributed microcracks arising at the boundaries between aggregate grains and the cement stone. In a certain weak zone of body, the microcracks will accumulate into a crack band, called fracture process zone, FPZ.

In the descending branch of the stress-strain line micro-cracking, especially within the FPZ, signifanctly increases. In the test, the strain is monotonically increased. The response to that is the steady decrease of "supportable" stress. We speak of strain-softening (reverse: strain hardening for steel). The message is: concrete is not totally brittle. Fig. 17 shows test results.

Enormereous research has been devoted to these phenomena and to the modelling of the entire σ-ε-line in tension. In this course, we shall use a simple approach. We stipulate ideal elasticity i.e. Hookean behaviour:

$$\sigma(\varepsilon) = \varepsilon \cdot E_{ct} = \varepsilon\, E(t_e) \qquad (42)$$

with: E_{ct}, from Eq.(34), (37) or (38), resp.. By neglecting the micro-cracking strain ε_r, we underestimate the concrete's ability to adapt to imposed strain by microplasticity.

Fracture occurs if σ reaches f_{ct} (normal stress failure criterion) at a strain

$$\varepsilon_u = f_{ct}(t_e)/E(t_e) \qquad (43)$$

For concrete in axial compression, we have already stipulated the equality of $E_c = E_{ct} = E(t_e)$. We may assume also a straight σ-ε-line in compression. For the computation of thermal stresses due to hydration, it suffices to say that $|\sigma| \le f_c(t_e)$.

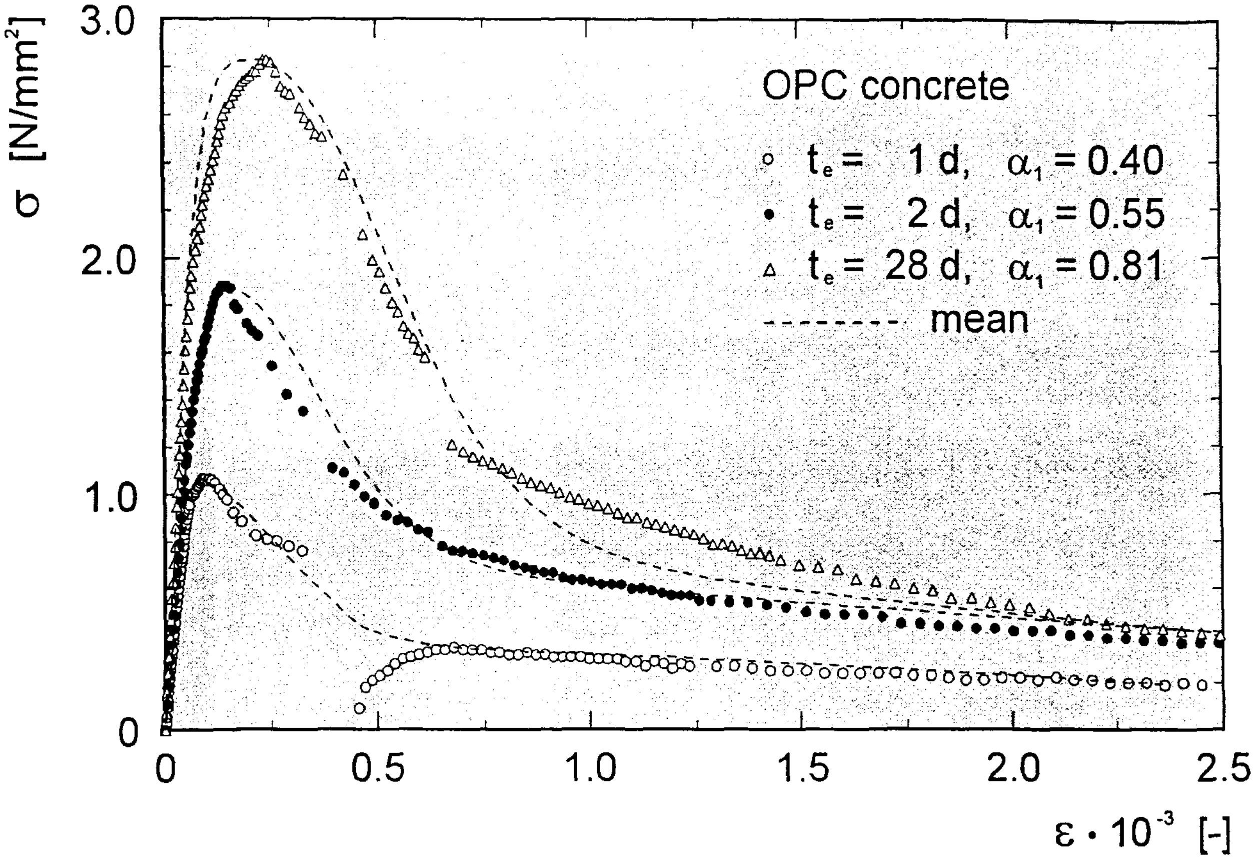


Fig. 17: Stress-strain lines in axial tension dependent on age loading - test results and model

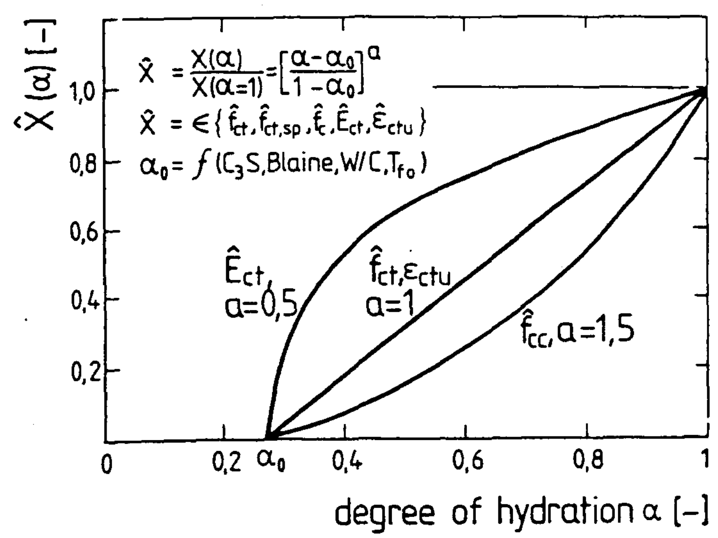


Fig. 18: Main mechanical properties vs. degree of hydration

5.5 Uncertainty of Models and Scatter

5.5.1 Comparison of models

In the previous sections we have already dealt with the models of the tensile and compressive strength and with that of Young's modulus. Fig. 18 shows a comparison of models which are deterministic and which describe the average behaviour. The well-known fact that the tensile strength grows faster than the compressive strength can be discerned. The Young's modulus grows fastest. The initial degree of hydration α_0 is a value found by regression of tensile test data, whereby a straight-line relationship was stipulated aforehand.

5.5.2 Model uncertainties

Although our models depict the test results well, they contain uncertainties, expecially for low degrees of hydration. This fact is shown by Fig. 19. The evolution of mechanical properties sets-on much earlier. Research is needed to improve the models for small degrees of hydration.

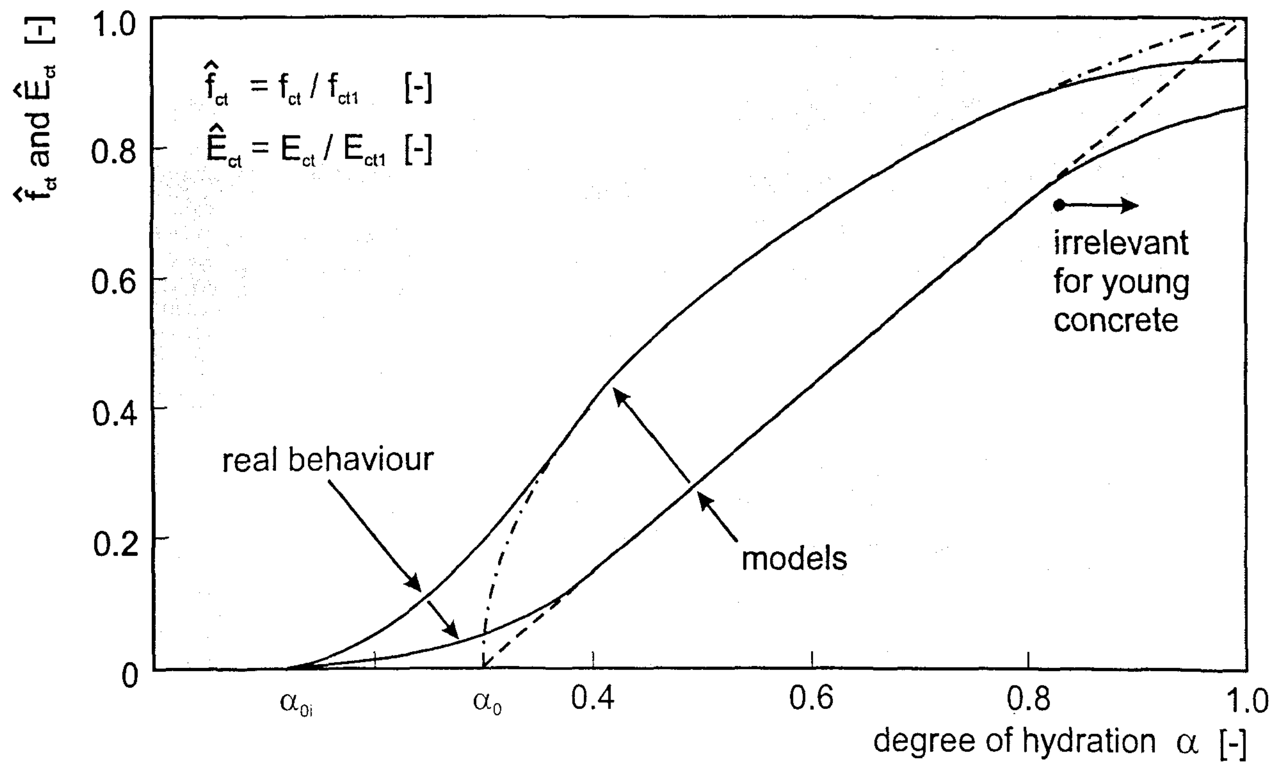


Fig. 19: Normalized tensile strength and modulus of elasticity dependent on degree of hydration - models and real behaviour

5.5.3 Scatter

The test data of the tensile and compressive strength, the Young's modulus and of other mechanical parameters exhibit scatter. Fig. 13 and 16 underline this fact. We found that the density distributions can for any α be described as normal Gaussian. This enables us to define relevant quantiles by

$$f_{i\,\%} = f_{50\,\%}\,(1 \pm 1.64\ V_i) \tag{44}$$

with:

$f_{50\,\%}$	average value of strength (most probable value)
$f_{i\,\%}$	quantile with i % probability of being exceeded or vice versa
(+)	for exeeding, sign in Eq.(44)
(-)	for falling below, sign in Eq.(44)
$V_i = \dfrac{\sigma}{f_{50\%}}$	coefficient of variation of property f
σ	scatter [N/mm^2]

We found the following coefficients of variation:

$V_{ct} = 0.15$; $V_c = 0.10$ and $V_E = 0.10$. For example with $f_{ct} \mathrel{\hat{=}} f_{ct50\,\%} = 30$ MPa

$f_{ct5\,\%} = 3.0\,(1 - 1.64 \cdot 0.15) = 2.26$ MPa 5 %-quantile

5.5.4 Relevant resistances for crack control

For the Young's modulus we may use the average value because, E is a property of the concrete member's entire cross-section.

For the tensile strength we should use a lower boundary value (beware of disappointment!); e.g. the 5 %-quantile. But this is not well thought unto the end, research is needed.

5.6 Creep

5.6.1 Necessity of modelling, pre-suppositions

The assessment of thermal stresses requires the realistic modelling of the viscoelastic behaviour of young concrete, especially in tension. Viscoelasticity encompasses the phenomena of creep and relaxation, both of them being significantly dependent on age and temperature history.

Eminent research effort to clarify these phenomena has been invested. It is within the frame of this course not possible to present the state of art. A rather condensed and simplified approach will be presented. The following pre-suppositions are made:

- linear viscoelasticity with aging is assumed
- creep fracture does not occur
- cracking strain ε_{cr} is neglected
- only basic creep is regarded (no drying of concrete)
- increase of elastic modulus during time under stress is neglected.

5.6.2 Creep function and superposition

Creep is the time-dependent and inelastic increase of deformation under long-term stess. With Fig. 20 we express the total mechanical strain of a concrete specimen at the age t, which was subjected to a constant stress at the age t_1:

$$\Delta\varepsilon(t - t_1; t_1) = \Delta\varepsilon_{el}(t_1) + \Delta\varepsilon_{cr}(t - t_1; t_1) \qquad (45)$$

We introduce the creep function φ to express the creep strain:

$$\Delta\varepsilon_{cr}(t - t_1; t_1) = \Delta\varepsilon_{el}(t_1) \cdot \varphi(t - t_1; t_1) \qquad (46)$$

with: $t - t_1$, time under stress; t_1, age at first loading. Furthermore:

$$\Delta\varepsilon_{el}(t_1) = \frac{\Delta\sigma(t_1)}{E(t_1)} \mathrel{\hat{=}} \frac{\Delta\sigma_1}{E_1} \qquad (47)$$

$$\Delta\varepsilon_{cr}(t - t_1; t_1) = \frac{\Delta\sigma_1}{E_1} \cdot \varphi(t - t_1; t_1) \qquad (48)$$

$$\Delta\varepsilon(t - t_1; t_1) = \frac{\Delta\sigma_1}{E_1} \cdot \left[1 + \varphi(t - t_1; t_1)\right] \tag{49}$$

Because the concrete is hardening and the temperature is variable, reference to real age at first loading is misleading. More correct in the introduction of the effective age t_{e1}. Thus, we obtain e.g. for the creep strain:

$$\Delta\varepsilon_{cr}(t - t_1; t_{e1}) = \frac{\Delta\sigma(t_{e1})}{E(t_{e1})} \cdot \varphi(t - t_1; t_{e1}) \tag{50}$$

As hardening proceeds, the creep function will change as concrete's viscose deformability diminishes. This fact is expressed by the spectrum of creep functions in Fig. 21: creep is aging.

If the stress varies with time, we apply the so-called principle of superposition to express the total strain at a certain age t_R. Thereby, the stress history $\sigma(t)$ is depicted by stress-steps $\Delta\sigma_i$ = const whose individual strain responses are summed up, Fig. 22:

$$\varepsilon(t_n - t_1, t_{e1}) = \sum_1^n i \frac{\Delta\sigma_i}{E_i}\left[1 + \varphi(t_n - t_i, t_{ei})\right] \tag{51}$$

5.6.3 Creep model of concrete in tension

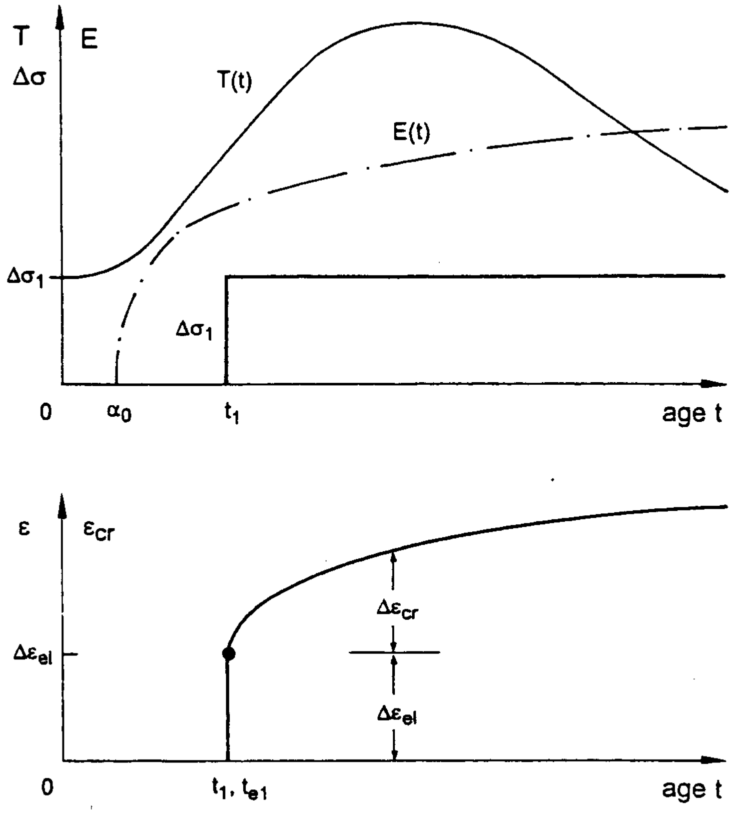
Creep of young concrete can be modelled in various ways. Well suited and comprehensively verified by tests is the following creep function:

$$\varphi(t - t_i, t_{ei}) = P_1(\alpha_i)\left(\frac{t - t_i}{t_c}\right)^{P_2(\alpha_1)} \tag{52}$$

with: t_c = 1 h; t - t_i, time [h] under stress; t_i, age at on-set of stress step $\Delta\sigma_i$ = const; t_{ei}, effective age as before, Eq.(14); α_i, degree of hydration as before for t_{ei}; P_1 and P_2, aging and shape parameters. Fig. 23 shows test results and their description by the model. The parameters P_1 and P_2 were for specific concrete determined with test data. E.g. for OPC-concrete with fly-ash, we found for $\alpha_0 < \alpha_i \leq 1$:

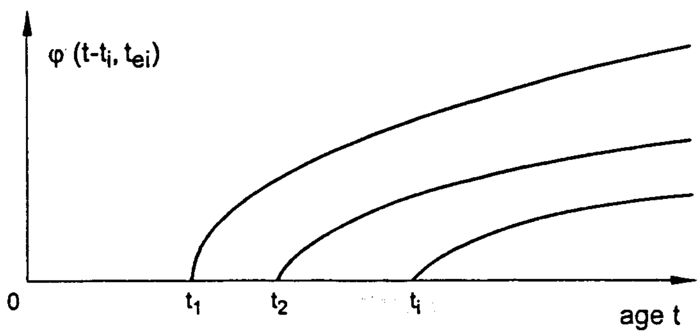
$$P_{1i} = 0.32 - 0.29\,\alpha_i \tag{53}$$

$$P_{2i} = 0.26 + 0.15\,\alpha_i \tag{54}$$



one-step creep, definitions, notations

Fig. 20



creep spectrum

Fig. 21

The value of α_i depends on t_{ei}, Eq.(18). Hence Eq.(52) can also be expressed in terms of t_{ei}. It can also be applied for creep of concrete in compression.

5.7 Relaxation

5.7.1 Necessity of modelling, definitions

The stresses arising during hardening of concrete are caused by imposed strains which are partly to totally restrained. The strains result from differences of temperature and of autogeneous shringage. If such imposed strains act for a long time, the stresses will be "auto-relieved" by relaxation.

Hence, relaxation is the long-term relieve of stress due to the viscose component of concrete's deformability. Fig. 24 shows the process. The strain-step $\Delta\varepsilon_1$ = const causes at $t_1(t_{e1})$ the stress-jump

$$\Delta\sigma(t_1) = \Delta\sigma_1 = \Delta\varepsilon_1 \cdot E_1 \tag{55}$$

From then on, relaxation begins. At the time t, the "residual" stress will be

$$\Delta\sigma(t - t_1, t_{e1}) = \Delta\varepsilon_1 \cdot E_1 \cdot \psi(t - t_1, t_{e1}) \tag{56}$$

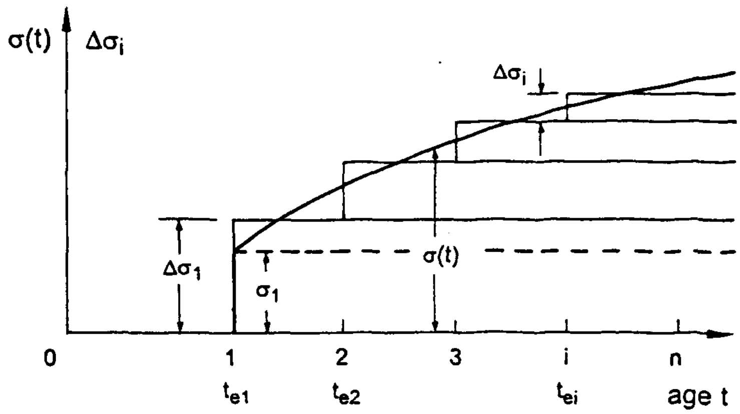
with: ψ, relaxation function; $t = t_1$; $\psi(0, t_{e1}) = 1$. The "loss" of stress is $\Delta\Delta\sigma_1$.

As for the creep function, the relaxation potential decreases with age. The relaxation spectrum can be expressed by $\psi(t - t_i, t_{ei})$, shown in Fig. 25.

If the imposed strain varies with time, we apply the principle of superposition to express the total stress at a certain age t_n. Thereby, the strain history $\varepsilon(t - t_1, t_{e1})$ is depicted by strain-steps $\Delta\varepsilon_i$ = const whose individual stress responses are summed-up, Fig. 26:

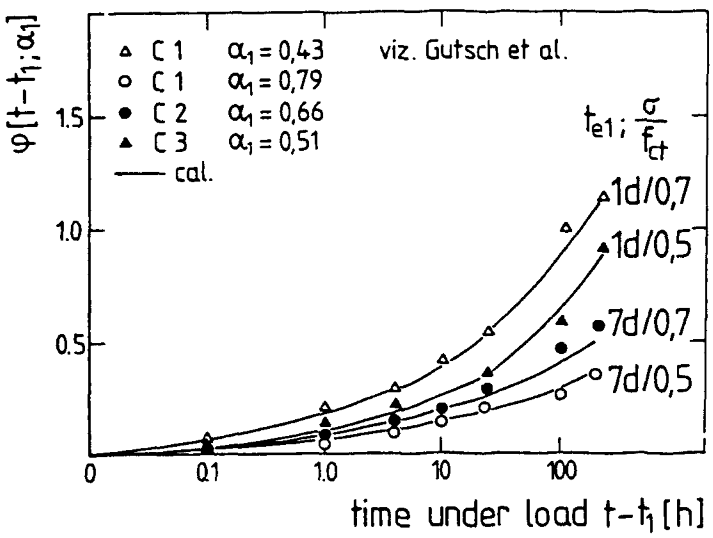
$$\sigma(t_n - t_1, t_{e1}) = \sum_1^n {}_i\, \Delta\varepsilon_i \; E_i \; \psi(t_n - t_i, t_{ei}) \tag{57}$$

If time increments Δt_i are small, the dashed envelope of stresses will be attained.



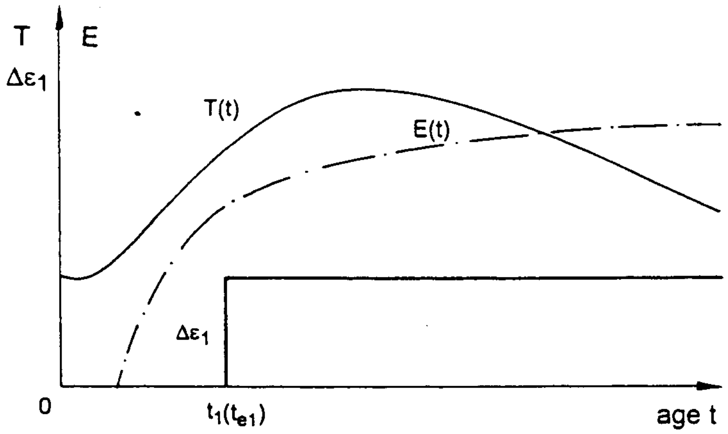
step-wise description
of stress history

Fig. 22



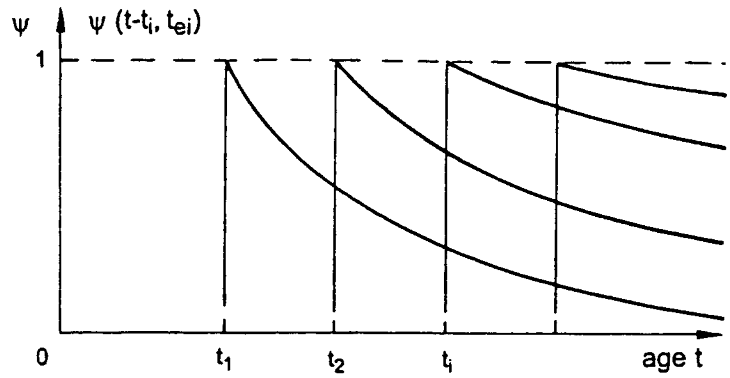
tensile creep dependent on
degree of hydration at loading

Fig. 23



one-step relaxation, definitions, notations

Fig. 24



relaxation spectrum

Fig. 25

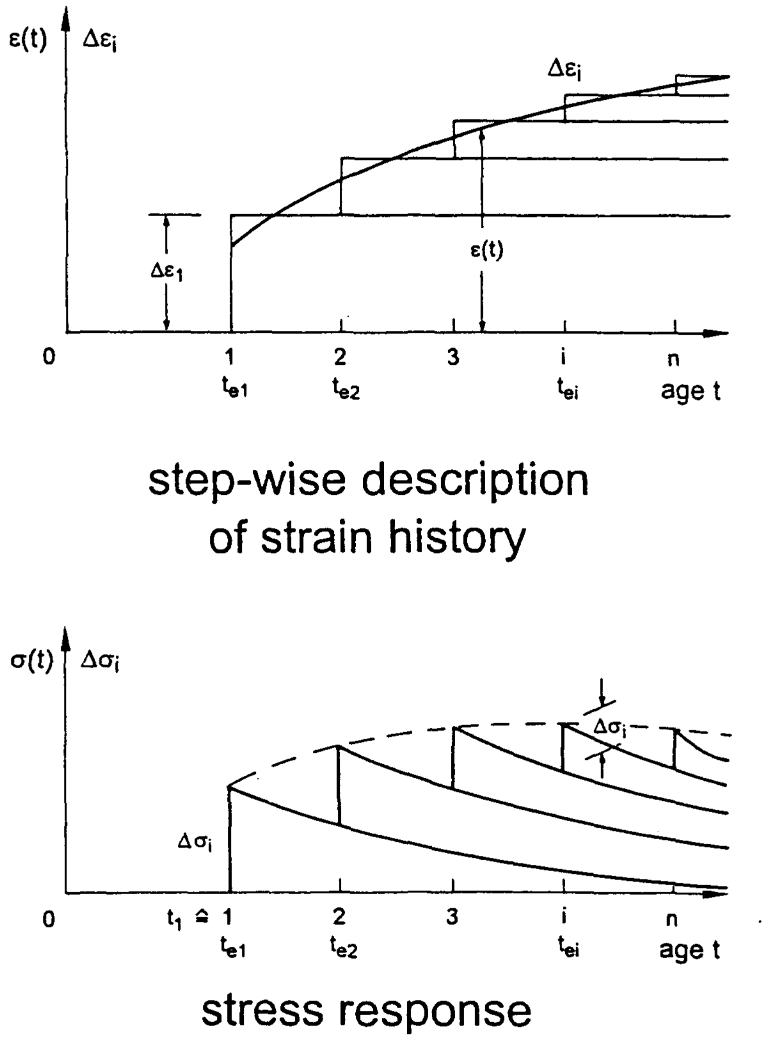


Fig. 26

Fig. 27 shows test results and relaxation functions calculated with Eq.(63).

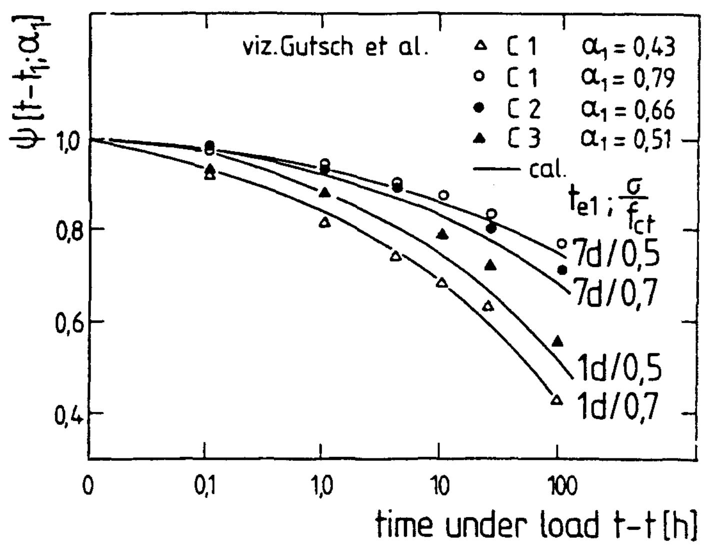


Fig. 27: Tensile relaxation dependent on degree of hydration at loading

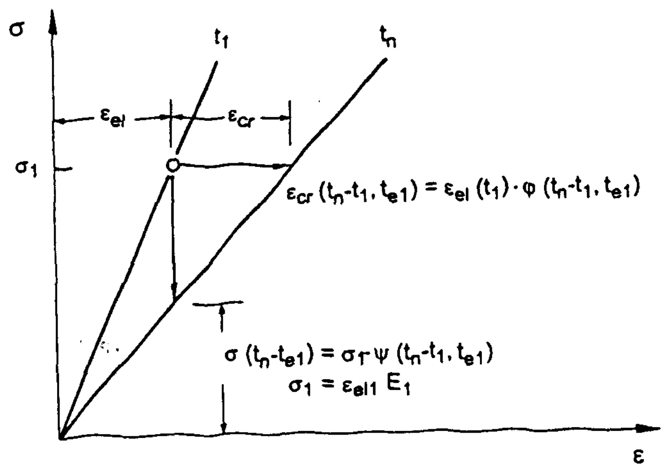
5.7.2 Relaxation model of concrete in tension

Creep and relaxation are related. Linear, non-aging linear visco-elasticity (LVE) expresses this relationship by:

$$\psi = \frac{1}{1+\varphi} \tag{58}$$

This relationship can be geometrically derived from Fig. 28. showing the creep isochrones for the time (t_i - t_1) under stress. The horizontal arrow is the creep strain, the vertical arrow corresponds to the stress-loss:

$$\Delta\sigma = \sigma_1 - \sigma_1\psi = \sigma_1(1-\psi) = \varepsilon_{e\ell 1}\ E_1(1-\psi) \tag{59}$$



relationship between creep and relaxation on basis of linear, non-aging viscoelasticity

Fig. 28

The stress at t_i is

$$\sigma_{i1} - \sigma_1 \psi = \varepsilon_{e\ell 1} \, E_1 \, \psi \tag{60}$$

Many tests have shown, that by such simple transformation of the creep function into the relaxation function the data test results could not be satisfactority described. The reason is aging, not taken into account by non-aging LVE. This deficit can be overcome by introduction of the aging parameter ρ_i

$$\rho_i = 0.55 + 0.6\,\alpha_i \leq 1;\ \alpha_0 < \alpha_i \leq 1 \tag{61}$$

With that, the relaxation function derived on basis of Eq.(52) is:

$$\psi = \frac{1 - (1 - \rho)\varphi}{1 + \rho\varphi} \tag{62}$$

or more precisely

$$\psi(t - t_i, t_{ei}) = \frac{1 - (1 - \rho_i)\ \varphi(t - t_i, t_{ei})}{1 + \rho_i\ \varphi(t - t_i, t_{ei})} \tag{63}$$

with: φ, acc. to Eq.(52).

6. FIELDS OF TEMPERATURES AND PROPERTIES OF CONCRETE

6.1 Problem and Objectives

The final aim of our efforts is the assessment of stresses in the hardening concrete member. For that, the computation of the fields of various properties, parameters etc. is essential. The starting point is the field of temperature $T_j(t)$ which varies in space and age. Accordingly, the fields of free (unimpeded) strain and curvature, of the degree of hydration and the mechanical properties must be generated.

Within the frame of this course, we shall deal with the practical case of the massive slab cast on ground. Emphasis will be put on the fields of temperature and degree of hydration. Ways to compute the fields of mechanical properties will elucidated.

6.2 Idealization of Problem and Differential Equation of that Conduction

If the dimensions of the slab in the groundplan are large in comparison with its depth d, then it can be geometrically idealized. It is acceptable to depict the slab by a strip with the cross-section of d x 1.0 m. The vertical dashed lines in Fig. 6.1 represent the planes of cut-out. There will be no heat flux through these planes. Hence, they are adiabatic boundaries. As the strip is regarded as infinitely long, there will also be no heat flux normal to plane d x 1.0 m. Heat flow will therefore be one-dimensional through the top and bottom face of slab. This is expressed in terms of temperature by $T_j(t) = T(z, t)$.

Heat flow is described by Fourrier's differential equation

$$\frac{\partial^2 T}{\partial z^2} \frac{\lambda_c}{c_c \, \rho_c} + \frac{q}{c_c \, \rho_c} - \frac{\partial T}{\partial t} = 0 \qquad (64)$$

with: q, source of heat of hydration (kJ/hm^3 or W/m^3; 1 kJ/hm^3 = 0.278 W/m^3; viz. sec. 3 of M and App. A). Because the source is non-linear in space and time, Eq.(63) can only be solved incrementally. For this course, the software ETAHB was written, which is attached as App. B. Numerical integration is there-in performed by the method of explicit differences.

6.3 Initial and Boundary Conditions

Initial temperatures of concrete and ground

In Fig. 6.1 the initial temperature values at t = 0 are shown: $T_c(z, 0) = T_{c0}$ and $T_s(z, 0) = T_{s0}$. Computation starts at the age t = 0. Although casting of concrete will consume a certain time, instantaneous casting is assumed.

Heat transfer

Heat transfer to ambiant air is described by the heat transfer coefficient α_e which globally encompasses several transfer phenomena. It mainly depends on the wind velocity. Heat transfer to air is modelled with Newton's boundary condition, Fig. 6.2:

$$\lambda_c \left(\frac{\partial T}{\partial z}\right)_S = \alpha_e (T_s - T_a) \qquad (65)$$

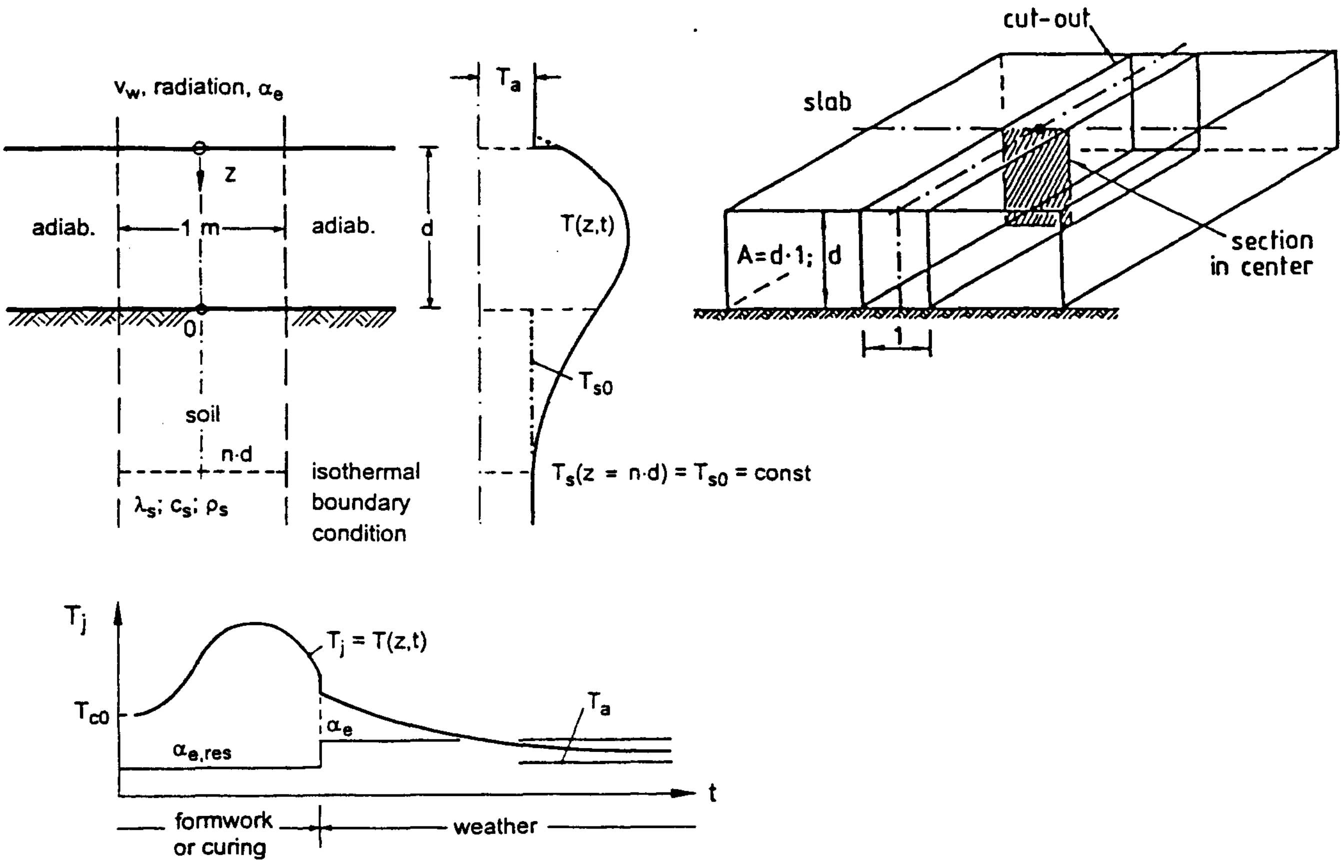


Fig. 6.1: Slab on ground, initial and boundary conditions

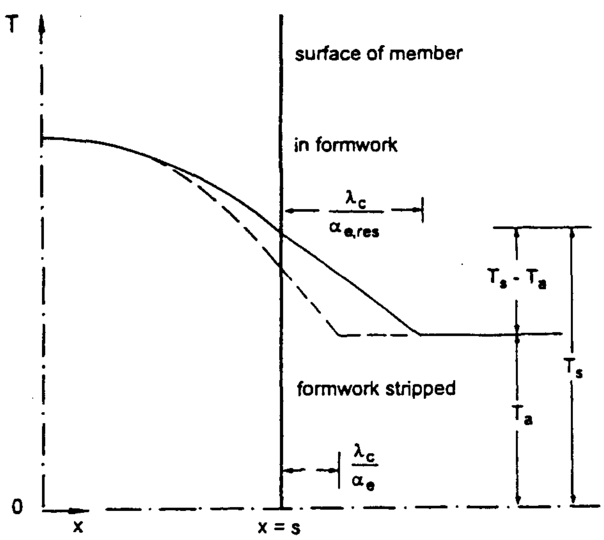


Fig. 6.2: Newton boundary condition

with: T_s, temperature of surface; T_a, temperature of air; s, denoting the flux vector normal to surface s. If the surface of cast concrete is covered for some time immediately after casting with a protective layer (e.g. PE-sheet in summer), then the resultant coefficient of heat transfer may include this effect

$$\frac{1}{\alpha_{e,res}} = \frac{1}{\alpha_e} + \frac{d_p}{\lambda_p}, \tag{66}$$

with: d_p, thickness and λ_p, coefficient of heat conduction of protective layer. After removal of layer, α_e will prevail. Eq.(66) can also be used to model the effect of formwork.

Environmental actions

These actions pertain to meteorogical influences and to the contact temperatures with adjoining solids and fluids. The influences of wind, radiation etc. are covered by the heat transfer coefficient α_e.

The temperature of air depends on many influences, especially on seasonal weather. Fig. 6.3 gives examples for a day in summer and in winter in 1997 in Berlin. One has to assume - for the period of casting and curing of member - a representative mean temperature T_a of day which is also the initial temperature $T_a = T_a(0)$. The fluctuations of air temperature during the day may be modelled by a sine-wave. It was found that the mean fresh concrete temperature can be expressed by T_a by

$$T_{c0} \approx 10 + 0.6\,(T_a + 5) \quad [°C]$$

The slab rests on dry to wet ground (sand/gravel). Heat flow will also occur through bottom face of slab (Fig. 6.1), the ground will be heated up. It is assumed that the temperature of ground will only be affected by heat flow within over a certain depth below bottom plane: $t \approx n \cdot d$, with $n \geq 4$. Hence, in the total depth from top of slab $z_{iso} \approx (n + 1) \cdot d$ an isothermal boundary with $T_s(z_{iso}, t) = T_{s0}$ = const may be assumed.

Values for computation

For the computation, a reasonable set of parameters, coefficients etc. is assumed which is shown in App. B of ETAHB.

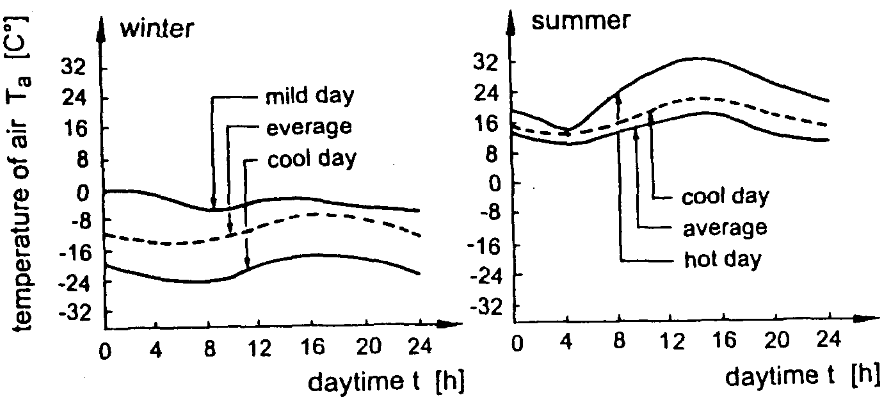


Fig. 6.3: Variation of temperature of air in summer and winter

6.4 Fields of Temperature, Thermal Strain and Curvature

For the standard concretes, the temperatures T(z, t) will be computed with ETAHB and presented dependent on t and t_e in tables. Free thermal strain is determined by

$$\varepsilon_0(z, t) = \alpha_T(T(z, t) - T_{c0}) \tag{67}$$

with: α_T, coefficient of linear thermal expansion; $T(z, t) - T_{c0} = \Delta T(z, t)$, temperature difference beyond placing temperature. For α_T, we assume

$$\alpha_T \approx 10 \cdot 10^{-6}\, K^{-1} \tag{68}$$

The free thermal strain varies with time and location just as T(z, t). The mean value of strain across depth of slab is expressed incrementally, Fig. 6.4:

$$\Delta\varepsilon_{0mi} = \frac{1}{\ell} \sum_{1}^{\ell}{}_k\, \Delta\varepsilon_{0ki}\,, \tag{69}$$

if the slab's thickness d is described by ℓ layers of the thickness dz = constant (e.g. d/10); and with i the on-set of strain step at t_i. The coordinate of center of the layer k from top of slab is z_k. We obtain the total free mean thermal strain by summation over time from t_1 to t_n:

$$\varepsilon_{0mn} = \sum_{i=1}^{n}{}_i\, \Delta\varepsilon_{0mi} \tag{70}$$

Because thermal strain is non-linear across depth, for later stress computation also the free thermal curvature $æ_0$ is important. Again, we express $æ_0$ incrementally:

$$\Delta æ_{0i} = \frac{\sum_1^{\ell} \Delta\varepsilon_{0ki} z_k}{\sum_1^{\ell} z_k^2} \qquad (71)$$

The autogeneous shrinkage can because of the rather high water-cement ratios of the standard concretes be suppressed.

6.5 Fields of Mechanical Properties

The values of temperature and degree of hydration as computed determined with ETAHB are printed out dependent on t and t_e. Hence, f_{ct}, f_c, E_{ct} can - with the formulae of sec. 4 - be generated dependent on α, t and t_e.

7. RESTRAINT, THERMAL STRESSES AND CRACKING

7.1 Causes and Kinds of Restraint

The major cause of early cracks in young concrete members is restraint. Restraint is the partial to total hindrance of the free thermal dilations. We may distringuish between two kinds of restraint: internal and external restraint.

Internal restraint is caused by the non-linear distribution of thermal strain across section, Fig. 6.1. Because of the requirement of "plane sections remain plane" (Bernoulli's law), thermal eigenstresses/-strains arise to ful-fill this requirement. In the general case, the resultant plane is defined by the mean strain ε_{0m} and the curvature $æ_0$. The eigenstresses self-equilibrate, no resultant moment and normal force exist.

External restraint is caused by the hindrance of thermal dilations of the structural member as a whole. The types of hindrance are manifold, e.g.:

- hindrance of thermal movement of slab due to shear-friction between slab and ground
- hindrance of thermal movement of young wall on old foundation
- hindrance of thermal movement of top slab of a tunnel by the old walls etc.

External restraint arises only in statically indeterminate structures. Restraint reactions must equilibrate among them. Internal restraint, being void of external hindrance, hardly ever occurs.

In massive structures, internal and external restraint co-exist. With concrete being a material with varying visco-elasticity across section, separation of stresses caused by these kinds of restraint is strictly not possible.

Hindrance of thermal movement can be partial or total. The stronger the hindrance, the higher the restraint reactions and the probability of severe cracking.

7.2 A Simple Case of Restraint

A simple case of restraint is shown in Fig. 7.1. The bar is axially restrained by an extensional spring on the right hand side. The free curvature $æ_0$ is not impeded. The axial deformation is hindered though not totally, depending on the spring stiffness c_s [mm/N].

The free thermal deformation of the bar amounts to:

$$\Delta\ell_0 = \varepsilon_{0m}L = \alpha_T\,\Delta T_m \cdot L \tag{72}$$

The deformation of spring, caused by the yet unknown restraint force N, is:

$$\Delta\ell_s = N \cdot c_s \tag{73}$$

and that of bar by N:

$$\Delta\ell_m = \frac{N}{E_e\,A} \tag{74}$$

with: A, cross-section of bar; E_e, effective (average) Young's modulus. Compatibility requires that

$$\Delta\ell_0 + \Delta\ell_{sp} + \Delta\ell_m = 0 \tag{75}$$

With that, we arrive at the axial restraint force:

$$N = -\alpha_T \, \Delta T_m \, E_e \, A \cdot \frac{1}{1 + \frac{E_e \, A \, c_s}{L}} \tag{76}$$

If the spring stiffness $c_s = 0$ [mm/N], total restraint exists. With $c_s > 0$, restraint diminishes. If $c_s \rightarrow \infty$, there is no restraint all, $N = 0$. Hence the term

$$R_N = \frac{1}{1 + \frac{E_e \, A \, c_s}{L}} \tag{77}$$

can be interpreted as axial restraint factor: $R_N = 1$, total restraint etc..

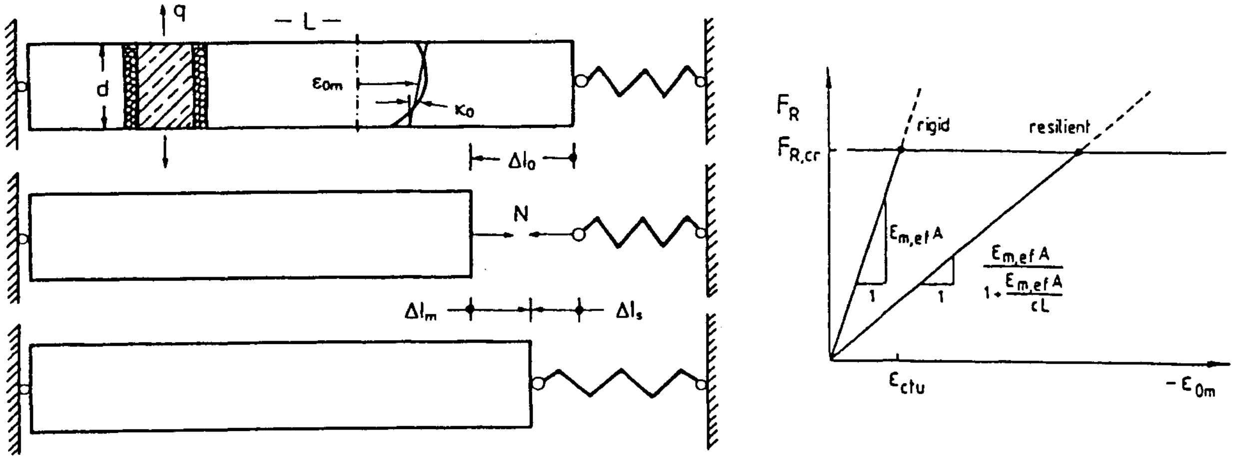


Fig. 7.1: Axial end restraint of a bar by a spring

If, in addition, also the end rotation $æ_0 L$ would be restrained by a rotational spring, we would obtain the restraint moment:

$$M = - æ_0 \, E_e \, I \, R_M \tag{78}$$

with the moment restraint factor:

R_M

$$\cancel{RM} = \frac{1}{1 + \frac{E_e \, I \, c_r}{L}} \tag{79}$$

Here-in are: I, moment of inertia; c_r, rotational spring stiffness in rad [-]/Nmm. Again, $R_M = 1$ with $c_r = 0$ etc.

The relationships contain several pre-suppositions:

- thermal deformation was described by the mean axial thermal strain ε_{0m} and $æ_0$ alone. Hence, the strain non-linearity across section - leading to self-equilibrating stress - was suppressed,
- the variation of thermal strain with time was not considered,
- in the effective Young's modulus, viscoelasticity is globally taken into account.

7.3 A More Concise Approach

For a linear, bar-type cut-out from a large slab as shown in Fig. 6.1, the distribution of free thermal strains is depicted in Fig. 7.2. We assume one-dimensional heat flow normal to bar's axis though top and bottom faces; side faces are adiabatic boundaries. Only longitudinal strains and stresses arise. The compensation plane of free dilation is determined by ε_{0m} and $æ_0$. External restraint will shift and rotate the compensation plane into the strain plane of resilient restraint. This we express by:

$$\varepsilon_{res} = \varepsilon_{0m} + \varepsilon_m \tag{80}$$

$$æ_{res} = æ_0 + æ_m \tag{81}$$

with:

ε_m restraint strain, assoc. with restraint stress

ε_{res} resultant strain

$æ_m$ restraint curvature, assoc. with restraint stress

$æ_{res}$ resultant curvature

Next, we discuss the Eq.(80) and (81) in terms of hindrance:

1. Entirely unrestrained member

As there are no restraint actions; we obtain because of $\varepsilon_m = 0$; $æ_m = 0$

$$\varepsilon_{res} = \varepsilon_{0m}$$

$$æ_{res} = æ_0$$

2. Total axial restraint, no bending restraint

$\varepsilon_{res} = 0$

$æ_{res} = æ_0$

3. No axial restraint, total bending restraint

$\varepsilon_{res} = \varepsilon_{0m}$

$æ_{res} = æ_0 + æ_m$

4. Optional axial and bending restraint

$\varepsilon_{res} = \varepsilon_{0m} + \varepsilon_m$

$æ_{res} = æ_0 + æ_m$

Correspondence to the axially restrained bar of Fig. 7.1 can be established: $\varepsilon_{0m} \hat{=} \Delta\ell_0$; $\varepsilon_m \hat{=} \Delta\ell_m$ and $\varepsilon_{res} \hat{=} -\Delta\ell_s$.

Model

The bar's depth d is subdivided into ℓ strips of equal thickness $\Delta z = d/\ell$ (ℓ, even number; e.g. 10). The time dependent local strain history is modelled by strain steps. Compatibility for the strain step $\Delta\varepsilon_{0,ki}$ = const of the strip k at the time i of its on-set requires:

$$\Delta\varepsilon_{e\ell,ki} + \Delta\varepsilon_{0,ki} - \Delta\varepsilon_{res,i} - \Delta æ_{res,i}\, z_k = 0 \qquad (82)$$

With $\Delta\varepsilon_{e\ell,ki}$ elastic strain response at sudden strain step. With Eq.(82) we express the force of strip k at time i:

$$\Delta F_{ki} = -\frac{d}{\ell}\,(\Delta\varepsilon_{0,ki} - \Delta\varepsilon_{res,i} - \Delta æ_{res,i}\, z_k)\, E_{ki} \qquad (83)$$

This force diminishes in course of time $t_n - t_1$ under stress due to relaxation:

$$\Delta F_{k,ni} = -\frac{d}{\ell}\,(\Delta\varepsilon_{0,ki} - \Delta\varepsilon_{res,i} - \Delta æ_{res,i}\, z_k)\, E_{ki}\, \psi_{k,ni} \qquad (84)$$

Now, by summation over the cross-section from 1 to ℓ and over time t_n - t_1 we obtain the restraint reactions:

$$N_{n,1} = -\frac{d}{\ell} \sum_1^n {}_i \sum_1^\ell {}_k \left(\Delta\varepsilon_{0,ki} - \Delta\varepsilon_{res,i} - \Delta æ_{res,i}\, z_k\right) E_{ki}\, \psi_{k,ni} \tag{85}$$

$$M_{n,1} = -\frac{d}{\ell} \sum_1^n {}_i \sum_1^\ell {}_k \left(\Delta\varepsilon_{0,ki} - \Delta\varepsilon_{res,i} - \Delta æ_{res,i}\, z_k\right) z_k\, E_{ki}\, \psi_{k,ni} \tag{86}$$

These equations can only be evaluated incrementally. Thereby, the impeding interaction - described by ε_{res} and $æ_{res}$ - with neighbors must be known. However, if we replace the variable modulus $E_{ki}\, \psi_{kni}$ by an effective modulus E_e = const, we arrive at the relationship of sec. 7.2 for the restrained bar.

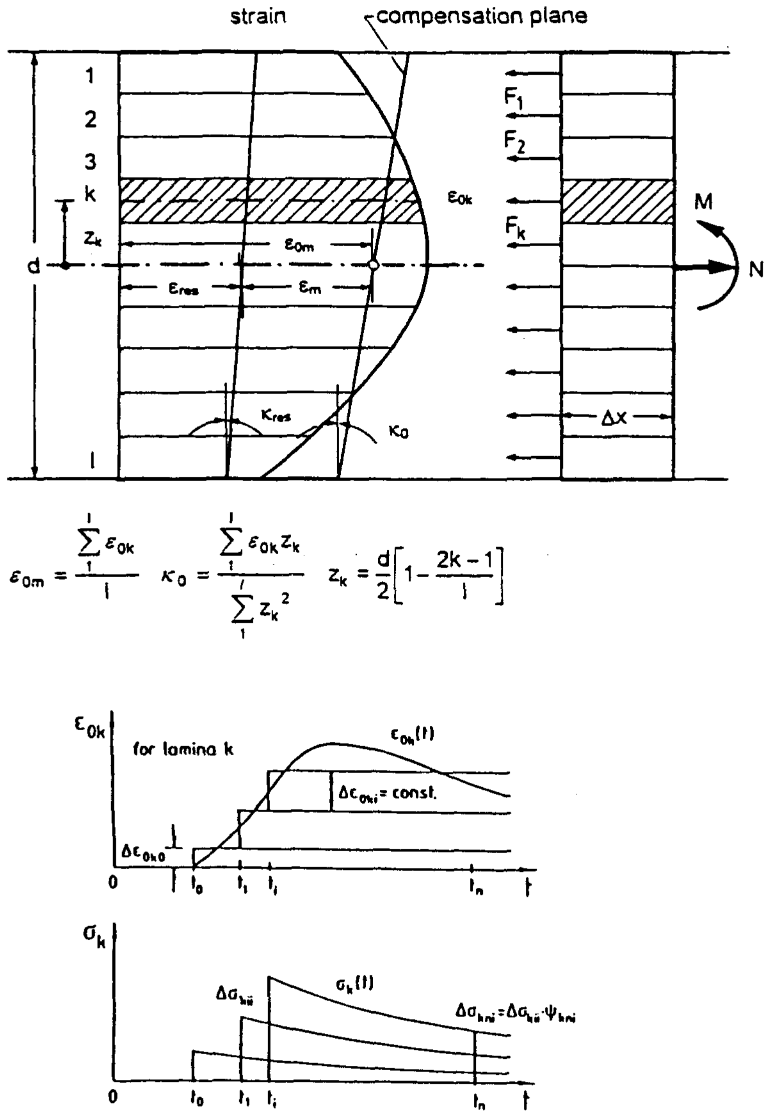


Fig. 7.2: Finite strip model, compatibility, strain history and stress response

7.4 Restraint of Slab on Ground

Shear friction

In Fig. 7.3 a 1 m wide cut-out from a slab with the thickness d, cast on sand/gravel ground, is depicted. Length of slab is L. Thermal contraction or expansion of slab is hindered by shear-friction interaction. This interaction can be described by a rigid-plastic bond stress/slip relationship

$$\tau_{fr} \approx \mu_{fr}\,\sigma_v \qquad (87)$$

independent on slip u. Here-in are:

$\mu_{fr} = \tan\varphi \approx 0.6$, friction coefficient,
$\sigma_v = d\rho_c$, vertical pressure on ground due to dead weight.

Irrespective of slip, the axial restraint force N_{fr} is built-up. It is associated with the moment $M_{fr} = N_{fr}\,d/2$. For axis of symmetry, we obtain the restraint actions

$$N_{fr} \approx \mu_{fr}\,d\rho_c \cdot \frac{L}{2} = \text{const} \qquad (88)$$

$$M_{fr} \approx N_{fr}\,d/2 \qquad = \text{const} \qquad (89)$$

which are constant over time $t_n - t_1$.

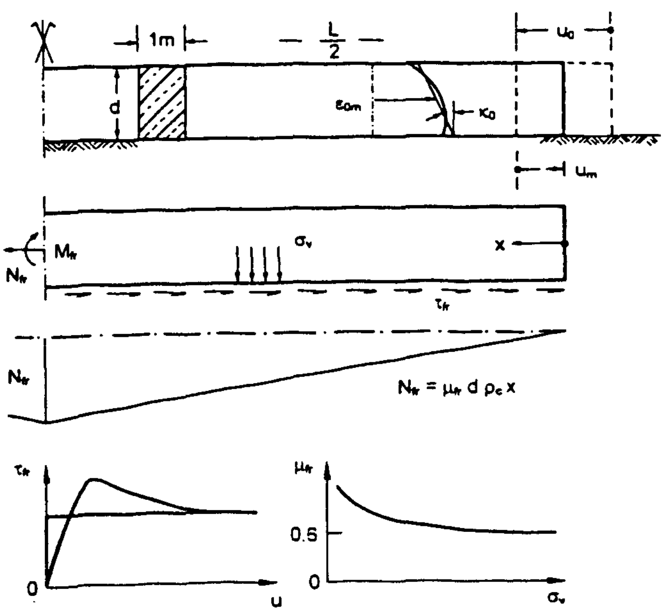


Fig. 7.3: Restraint of slab on ground by friction

Restraint conditions

The non-uniform thermal strain across depth d is associated with the mean thermal strain ε_{0m} and $æ_0$. If the curvature $æ_0$ were fully unhindered, the slap would tend to warp. This is for a thick slab unthinkable. We may assume that the slab cannot warp at all. Hence, total bending restraint with $æ_{res} = 0$ is relevant.

The axial restraint force is known by Eq.(88). Because it is constant, Eq.(85) must not be evaluated. The restraint force, however, is associated with a time-dependent axial strain ε_m as Young's modulus increases. This fact has to be taken into account for the restraint moment, Eq.(86), applying condition No. 4:

$$M_{n,1} = -\frac{d}{\ell} \sum_1^n i \sum_1^\ell k \left(\Delta\varepsilon_{0ki} - \Delta\varepsilon_{0m,i} - \frac{N_{fr} \cdot \ell}{E_{mi} \cdot d} \right) z_k \, E_{ki} \, \psi_{kni} \tag{90}$$

For evaluations in the pre-planning stage, we may replace the local relaxed modulus $E_{ki}\ \psi_{kni}$ by the average effective modulus of the $E_{ki}\ \psi_{kni} = E_{me,i}$.

Hence, we obtain

$$M_{n,1} \approx -\frac{d}{\ell} \sum_1^n i \, E_{me,i} \sum_1^\ell k \left(\Delta\varepsilon_{0ki} - \Delta\varepsilon_{0mi} - \frac{N_{fr} \cdot \ell}{E_{mi} \cdot d} \right) z_k \tag{91}$$

Because of

$$\sum_1^\ell k \, z_k = 0,$$

the second and third term in the bracket of Eq.(91) vanish. The ordinate z_k can be expressed by:

$$z_k = \frac{d}{2\ell} (\ell + 1 - 2\,k) \tag{92}$$

If this expression is introduced into Eq.(86), we obtain:

$$M_{n,1} \approx -\frac{d^3}{12} \sum_1^n i \, E_{me,i} \cdot \Delta æ_{0i} = -\ \frac{\widetilde{E}_{me,n1}\, d^3}{12}\ æ_{0n1} \tag{93}$$

This formulation is well-known from elasto-statics of beams. The evaluation Eq.(93) is performed as follows:

- Determination of mean Young's modulus across section dependent on time
  With the mean degree of hydration α_m over section with ETAHB, we obtain with Eq.(36), $E_m(\alpha)$, $E_m(t_e)$ and $E_m(t)$.

- Relaxation function ψ is approximated in steps by
  $\psi_m \approx 0.50$ for t = 12 to 48 hours
  $\psi_m \approx 0.70$ for t = 49 to 96 hours
  $\psi_m \approx 0.90$ for t = 97 to t_n hours (1000 h)

- Relaxed mean effective modulus is $\widetilde{E}_{m,en} \approx E_m(t) \cdot \psi_m(t)$

- Free thermal curvature is

$$\Delta æ_{0i} = \frac{\sum_{1}^{\ell} \Delta\varepsilon_{0ki}\ z_k}{\sum_{1}^{\ell} z_k^2} \tag{94}$$

and

$$æ_{0n1} \approx \sum_{1}^{n} i\ \ \Delta æ_{0,i} \tag{95}$$

Stresses and Cracking

Because of total moment restraint, cracks are expected to arise either on the upper or bottom face of slab. The edge stress is:

$$\sigma_{t,b} = \frac{N_{fr}}{d} \pm \frac{M_{n,1}}{d^2}\, 6 \tag{96}$$

Cracking will occur if

$$\sigma_{t,b} \geq f_{cte} \tag{97}$$

with: f_{cte}, effective tensile strength of the strips 1 or ℓ dependent on age acc. to Eq.(33) and (34). In order to take of the scatter of tensile strength into account, a certain reduction of strength is recommended, e.g.:

$$\hat{f}_{cte} \approx 0.75\ f_{cte}.$$

Counter Measures

If these approximative computations warrant excessive and deep cracks, the engineer has to deliberate and propose counter-measures. These might cover a large span of possibilities (some of them will be dealt with in the course). The most effective counter-measure aims at the reduction of heat release (reduction of T_{c0}, reduction of C-content, partial replacement of cement by FA or GBFS, cooling etc.).

Cracking and Reinforcement

Cracking is a normal thing in reinforced concrete. Completely undesireable are through-cracks which sever the entire member. They should be avoided, especially if ground water may enter into interior of structure or if water, oil, etc. stored in the structure may seep through the cracks. Members which inspite of cracks still exhibit a compression zone of tight concrete, are usually impervious.

Reinforcement cannot avert cracking. The width of cracks, however, can be limited., a costly way to achieve water-tightness (lim w = 0.1 mm).

Cracks can be sealed by resins, again a very expensive procedure.

RECOMMENDED READING

- Papers by N. Carino, viz. App. A
- ACI Manual of Concrete Praxis, latest ed., parts 1 and 3, various recommendations on mass concrete
- ASTM Codes C 595 M-95a and C 150-95a on cements OPC and blended cements GBFC.
- check ACI Materials Journals; e.g. May-June 1998.
- check ASCE Structural, Materials in Civil Engineering and Engineering Mechanics Journals
- www.aci-int.org

[1] MIT-2.DOC, 26.08.98 18:17

APPENDIX A

RELATIONSHIPS OF THE RELEASE OF HEAT OF HYDRATATION

1 TEMPERATURE OF FRESH CONCRETE T_{c0}

1.1 On Basis fo Temperature Values of Components in Execution Phase

$$T_{c0} = \frac{A \cdot c_A \cdot T_A + W \cdot c_W \cdot T_W + C \cdot c_C \cdot T_C + FA \cdot c_{FA} \cdot T_{FA}}{A \cdot c_A + W \cdot c_W + G \cdot c_G + FA \cdot c_{FA}} , \quad (1.1)$$

with the following masses fo concrete components:

A: total aggregate content (dry mass), kg/m^3,

C: cement (for all cements), kg/m^3,

W: total water content, kg/m^3,

FA: fly ash, kg/m^3,

c_i: specific heat capacity, kJ/(kg K), acc. to Table 2.1.

T_c etc. initial temperatures at time of mixing, °C

1.2 Assumptions for the Pre-Planing Phase

We assume for our computations the following values dependent on season:

Spring and Fall: T_{c0} = 15 °C

Summer: T_{c0} = 25 °C

Winter: T_{c0} = 10 °C

[1] **M** refers to manuscript

2 SPECIFIC HEAT CAPACITY

2.1 On Basis of Concrete Composition in Execution Phase

Tab. 2.1: Specific heat capacity and density of components

component	unit	water W	cement C	natural aggregate	fly ash FA
c_i	kJ/(kg K)	4.18	0.80	0.80	0.75
ρ_i	kg/m^3	1.0	3.0 - 3.1	2.6 - 2.8	2.4

specific heat capacity of concrete:

$$c_c = \frac{1}{\rho_c}\left(C \cdot c_C + W \cdot c_W + G \cdot c_G + FA \cdot c_{FA}\right) \quad . \tag{2.1}$$

2.2 Assumptions for Pre-Planing Phase

$0{,}9 \leq c_c \leq 1{,}00$ [kJ/kgK] for hardened concrete and usual moisture

$1{,}0 \leq c_c \leq 1{,}15$ [kJ/kgK] satuated and fresh concrete

1 kJ/kgK $\equiv$ 0.2778 Wh/kgK

We may assume here: c_c = 0.300 Wh/kgK and ρ_c acc. Tab. 7.1.

3 EFFECTIVE AGE OF CONCRETE

$$t_e = \int_0^t \exp \frac{E}{R}\left[\frac{1}{293} - \frac{1}{273 + T(t)}\right] dt \tag{3.1}$$

or expressed by summation (viz. **M**[1])

with:

T(t): temperature of concrete in °C at any point in structure

R: universal gas constant, R = 8.315 J/mol K

E: activation energy, T > 20 °C: E(T) = 33.5 kJ/mol

T ≤ 20 °C: E(T) = 33.5 + 1.47 · (20 - T) kJ/mol

273 K $\hat{=}$ 0 °C

293 K $\hat{=}$ 20 °C

4 DETERMINATION (ESTIMATION) OF max ΔT_{ad}

4.1 Determination on Basis of Analysis of Cement, GBFS, Hydraulic Binders and Known Composition (Execution Phase)

$$\max \Delta T_{ad} = \frac{C \cdot \left[(1 - m_{SL}) \cdot \sum m_i \cdot Q_i + m_{SL} \cdot Q_{SL}\right] + FA \cdot Q_{FA}}{c_c \cdot \rho_c} \tag{4.1}$$

with:

C cement content, kg/m^3

FA fly ash content, kg/m^3

m_{SL} = SL/C [-], normalized slag content

m_i = M_i/C [-], normalized content of clinker phase M_i from Bogue calculation

Q_i specific heat content of clinker phase, kJ/kg

Q_{SL} specific heat content of ground blast furnace slag, kJ/kg

Q_{FA} specific heat content of fly ash, kJ/kg

c_c, specific heat capacity of concrete, kJ/(kg K), viz. sec. 2

ρ_c, density of concrete, kg/m^3, viz. sec. 2, Table 7.1

Tab. 4.1: Specific heat of hydration Q_i of clinker phases and other components in kJ/kg

Stoffe	Q_{C3S}	Q_{C2S}	Q_{C3A}	Q_{C4AF}	$Q_{fr.\,CaO}$	$Q_{fr.MgO}$	Q_{SL}	Q_{FA}
Q_i [kJ/kg]	500	250	1340	420	1150	840	290	35

Determination of Reacting Phases - Bogue-Method

$m(C_3S)$ = 4.0710 CaO - 7.6024 SiO_2 - 1.4297 Fe_2O_3 - 6.7189 Al_2O_3 - 2.852 SO_3 (4.1)

$m(C_2S)$ = 2.8675 SiO_2 - 0.7544 C_3S (4.2)

$m(C_3A)$ = 2.6504 Al_2O_3 - 1.6920 Fe_2O_3 (4.3)

$m(C_4AF)$ = 3.0432 Fe_2O_3 (4.4)

constituents of pc-clinker in mass-percent without correction for loss of ignition.

4.2 Estimation of max ΔT_{ad} in the Pre-Planning Phase

4.2.1 On the Basis of German Experience

In the pre-planning/bidding phase, we have to make assumptions depending on strength class, dimensions of member etc. These assumptions pertain to the type of cement, its amount, to the amount of fly ash, etc. Tab. 4.2 presents average values of total heat of hydration of German cements of strength class 32.5 MPa applied for massive r/c-elements

Tab. 4.2: Heat of Hydration of German Cements (Average Values)

No.	Cement type	denomia-tion	type of release	for season	max $\hat{Q}_c$ [kJ/kg]	content 1) of GBFS [m.-%]	cement content C [kg/m³ concrete]
1	CEM I 32.5 R	OPC	strong heat	winter, spring, fall	480 - 520	0	≥ 270
2	CEM II/B 32.5 R	GBFS-C	medium heat	spring, fall	450 - 460	> 21 ≤ 35	"
3	CEM III/A 32.5	GBFS-C	low heat	summer	370 - 410	> 36 ≤ 65	"
4	CEM II/B 32.5	GBFS-C	very low heat	hot summer	360 - 365	> 66 ≤ 80	"

1) of total cement

If no fly-ash is added, max ΔTad can be determined with Eq. (4.1), FA = 0

$$\max \Delta T_{ad} = \frac{C \max \hat{Q}_C}{c_C \, \rho_C} \tag{5.1a}$$

If fly ash is added, Eq. (4.1) can be applied. Fly-ash content varies in practice between 60 and 120 kg/m^3.

4.2.2 Japanese Method

In the Japanese Standard Spec. of Design and Construction of Concrete Structures 1995, Part 2, the maximum adiabatic temperature rise

$$\max \Delta T_{ad} = \frac{\max Q_c}{c_c \ \rho_c} \ [K] \qquad (5.1b)$$

is expressed by

$$\max \Delta T_{ad} = a\,C + b \ [K] \qquad (5.1b)$$

with: C cement content, kg/m^3

T_{c0} fresh concrete temperature, °C

a parameter in K/kg

b parameter in K

Table 4.3: Japanese Method for Estimation of max ΔT_{ad}

No.	Cement type	T_{c0} [°C]	a [K/kg]	b [K]
1	OPC	10 20 30	0.12 0.11 0.11	11 13 12
2	moderate heat PC	10 20 30	0.11 0.10 0.11	6 9 9
3	fly-ash-PC 1)	10 20 30	0.15 0.12 0.11	-3 8 11
4	GBFS-PC 2)	10 20 30	0.11 0.10 0.11	14 15 15

1) $\frac{FA}{PC + FA} = 0.20$ 2) $\frac{SL}{PC + SL} = 0.40$

5 DEGREE OF HYDRATION FROM ADIABATIC TEST

5.1 Determination on Basis of Adiabatic Test Results

General:

$$\alpha(t) = \frac{Q\big(T(t)\big)}{\max Q}$$

with:

$Q(T(t))$ total heat release along temperature path $T(t)$, kJ/m^3 concrete,

$\max Q$ total heat release (potential) of concrete caused by cement plus other hydraulic binders, e.g. FAfor $t \to \infty$

$$\max Q \equiv \max Q_{ad} = c_c\, \rho_c\ \max \Delta T_{ad}$$

Adiabatic test:

$$\text{meas}\,\alpha(t_e) = \frac{c_c\, \rho_c\, \Delta T_{ad}(t)}{\max Q} = \frac{\Delta T_{ad}(t)}{\max \Delta T_{ad}} \tag{5.1}$$

Modelling:

With transformation of time t into effective age t_e with Eq. (3.1), the degree of hydration can be modelled with the JONASSON-equation:

$$\alpha(t_e) = \exp\left(-\left[\ln\left(1+\frac{t_e}{t_k}\right)\right]^{c_1}\right) \quad . \tag{5.2}$$

with c_1 and t_k parameters deduced from the measured values (ΔT_{ad}, T_{c0}, t) of adiabatic test by regression.

5.2 Estimation of Degree of Hydration in Pre-Planning Phase

5.2.1 Estimation on Basis of German Experience (Test Work)

The Jonasson parameters c_1 [-] and t_k [h] can roughly estimated for OPC-concrete (with a fly-ash content $\leq$ 20 %) dependent on the cement content

$$t_k \approx 4.92 + C \cdot 0.027 \tag{5.3a}$$

$$c_1 \approx -1.86 + C \cdot 0.0027 \tag{5.3b}$$

for $250 \leq C \leq 400$ kg/m^3.

5.2.2 Japanese Method to Estimate the Degree of Hydration

In the ISCE standard, cited in sec. 4.2.2, the time dependence of the adiabatic rise and degree of hydration is presented

$$\frac{\Delta T_{ad}(t)}{\max \Delta T_{ad}} = \alpha(t) = 1 - \exp(-r\,t) \qquad (5.3b)$$

with: $r = g \cdot C + h$; t, real age in calorimeter test in days; C cement content, kg/m^3

Table 5.1: Estimation of Degree of Hydratition During Adiabatic Test, acc. JSCE

No.	cement type	T_{C0} [°C]	g [m^3/kg d]	h [d^{-1}]
1	OPC	10 20 30	0.0015 0.0038 0.0040	0.135 -0.036 0.337
2	moderate heat PC	10 20 30	0.0003 0.0015 0.0021	0.303 -0.279 0.299
3	fly ash PC [1)]	10 20 30	0.0007 0.0028 0.0030	0.141 -0.143 0.059
4	GBFS-PC [2)]	10 20 30	0.0014 0.0025 0.0035	0.073 0.207 0.332

1) $\frac{FA}{PC + FA} = 0.20$ 2) $\frac{SL}{PC + SL} = 0.40$

For practical use, the age t in the adiabatic test has to be transformed into the effective age te with Eq. (14) of M in order to obtain $\alpha(t_e)$.

5.2.3 Danish Method

The Danish Method expresses the degree of hydration by

$$\alpha(t_e) = \exp\left(-\left[\frac{t_e}{t_k}\right]^{-b}\right) \qquad (5.3b)$$

with t_e, effective age in [h], $0.4 \leq b \leq 0.9$ [-], $20 \leq a \leq 40$ [h]

5.2.4 Shrinkage core model (SCM)

$$\alpha(t_e) = \frac{T_{ad}(t_e)}{\max \Delta T_{ad}} = \frac{a(t_e - t_0)}{1 + a\,(t_e - t_0)} \tag{8.3}$$

a: parameter [1/h]

t_0: end of dormant phase [h]

t_e: effective age [h]

References:

[1] Carino, N.J.: The Maturity Method: Theory and Application, Cement, Concrete and Aggregates, ASTM, pp. 61-73, 1984.

[2] Carino, N.J., Tank, R.C.: Maturity functions for concretes made with various cements and admixtures, ACI Materials Journal, pp 188-196, March-April 1992.

6 HEAT SOURCE FOR CALCULATION OF TEMPERATURE FIELD

At a point $j \mathrel{\hat{=}} (x,y,z)$ in the structure we obtain

$$q_j(T(t)) = \frac{dQ}{dt} = \max Q \frac{d\alpha}{dt} = \max Q \frac{\partial \alpha}{\partial t_e} \frac{\partial t_e}{\partial t} \tag{7.1}$$

with $q_j(t)$, heat source in $kJ/(m^3h)$ if t, t_e are counted in hrs.

7 CONCRETE COMPOSITIONS AND OTHER DATA

7.1 Standard Concretes

For the computation of the fields T(z,t), α(z,t) in the system massive slab on ground, the standard Concretes CO1, CO2 and CO3 will be used. Their compositions are given in Table 7.1. In Table 7.2 other relevant parameters are listed.

7.2 Data Describing Hydration

Chemical composition of the constituents of PC-clinker of the cements of Table 7.1 are listed in the Tables 7.3a and b.

Table 7.3a: Constituents of PC-Clinker of OPC for CO1 and CO3

for concrete	CEM I 32.5 R OPC	constituents of PC-clinker	m_i m.-%
CO1 and CO2	OPC	CaO	64.4
	100 %	SiO_2	21.5
	PC-clinker	Fe_2O_3	1.5
		Al_2O_3	5.6
		SO_3	3.0

mi, Q_i of OPC 270 kg/m^3, FA = 60 kg/m^3 with Q_{FA}, viz. Table 4.1 and Bogue calculation

Table 7.3b: Constituents of PC-Clinker of PC for CO2

for concrete	CEM III/B 32.5 GBFS-PC	constituent of PC clinker	m_i m.-%
CO2		CaO	66.3
		SiO_2	21.6
	PC-clinker	Fe_2O_3	2.1
		Al_2O_3	5.4
		SO_3	1.6

Cement constists of 32 m.-% PC clinker and of 68 m.-% slag. Specific heat of slag and of clinker phases are shown in Table 4.1. Calculation of clinker phases are given in sec. 4.1.

Tab. 7.1b: Standard Concrete Mixes

MIX No.	CO1	CO2	CO3
strength class	C 25	C 35	C 25
constituents	PC-concrete	GBFS-concrete	PC-concrete
type of cement	CEM I 32.5 R	CEM III/B 32.5	CEM I 32.5 R
cement [kg/m^3]	270	390	280
fly ash [kg/m^3]	60	-	220
water [kg/m^3]	175	183	197
W/(C + 0,3 FA)	0.61	0.47	0.57
aggregate			
0/1 [kg/m^3]	92.8	179.3	-
0/2 [kg/m^3]	556.7	484.1	-
2/8 [kg/m^3]	554.6	321.5	-
8/16 [kg/m^3]	644.6	800.8	-
sum [kg/m^3]	1848.7	1785.7	1828.0
admixture			
plasticizer [kg/m^3]	2.70	1.95	1.50
retard. [M.-% v. C]	-	-	0.4
density of fresh concrete ρ_c [kg/m^3]	2360	2360	2336

Tab. 7.1b: Parameters of Standard Concrete Mixes

MIX No.		CO1	CO2	CO3
strength class		C 25	C 35	C 25
constituents		PC-concrete	GBFS-concrete	PC-concrete
type of cement		CEM I 32.5 R	CEM III/B 32.5	CEM I 32.5 R
c_1	[-]	-1.13489	-1.18937	-1.08
t_k	[h]	12.0548	16.19731	26.8385
α_0	[-]	0.1995	0.3592	0.2462
f_{ct1}	[N/mm^2]	3.005	3.368	3.31
f_{c1}	[N/mm^2]	47.89	57.123	55.21
E_{ct1}	[N/mm^2]	33630	35717	36386

APPENDIX B

ETAHB

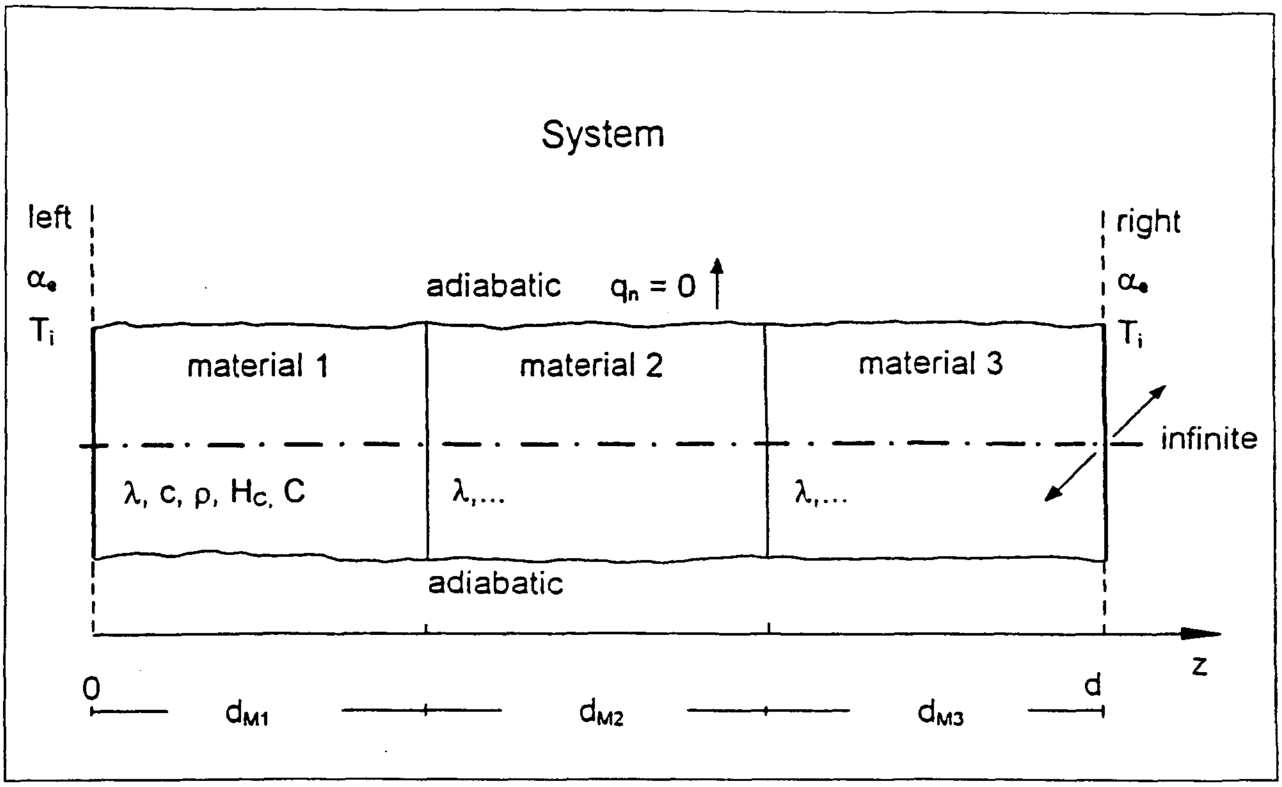


Fig. 1

1 ONE-DIMENSIONAL TEMPERATURE ANALYSIS OF HARDENING CONCRETE ELEMENTS WITH THE METHOD OF EXPLICIT DIFFERENCES

1.1 General Remarks

The program *ETAHB* was convinced to compute the temperature field, field of equivalent age and field of degree of hydratarion in a three layered body. Fig. 1 shows the definition of system.

The entire system consists of three subsystems of the materials M_1 to M_3. The coordinate *z* starts at the left edge of material M_1. At the left and right boundary, time-dependent boundary conditions can be chosen.

1.2 Systems Parameter

For the temperature analysis, the geometry, the materials, the initial and boundary conditions have to be stipulated. Also the time of computation has to be chosen.

1.3 Geometry

The thickness d of the entire system is the sum of the individual thicknesses d_{M1}, d_{M2} and d_{M3}. All of them greater than zero.

The program computes the field of temperature and the degree of hydratation at chosen coordinates z_k dependent on age t with the method of explicit differences. The values of z_k are chosen equidistant. They are in the center of the elements strips with the breath dz. The number of supports n is determined by

$$n = d / dz.$$

z_k is given by

$$z_k = (k\text{-}0.5)\ dz, \qquad k = 1,\ldots,n.$$

1.4 Materials

For each of the three materials, the material number, material name (both optional), the specific heat capacity c, the coeffizient of heat conduction λ and the density ρ have to be chosen. If one of the materials is hardening concrete, the maximum heat of hydratation of concrete $maxQ$, the total amount of cement C and the parameters t_k and c_1 of JONASSON'S formulation of degree of hydratation have to be stipulated. Other hydralic binders are taken into account.

1.4.1 Initial and Boundary Conditions

At the start of computation, $t = 0$, the initial temperatures T_0 of the materials must be chosen. Also at the boundaries $z = 0$ and $z = d$ the ambient temperatures T_a and the heat transfer coefficients α_e must be chosen. The ambient temperatures are constant. The coefficients of heat transfer can be defined in two consecutive time intervals.

1.4.2 Period of Time

The period of time t is divided into m equidistant intervals dt. dt is chosen to $1/360\ h$, so m can be calculated to

$$m = t\ 360.$$

The results are stored hourly.

2 EXAMPLES

2.1 Working with the program

The handling of the program is simple. Assume the example “name“ should be examinated, the program call is

ETAHB NAME <ENTER>.

One set of data consists of two input sets and three output sets (see next table).

Table 2-1: data set

FILES	INPUT	OUTPUT
”name.dat”	• geometry of system • time of analysis • discretisation of structure • boundary conditions	-
”name.mat”	• material values	-
”name.tem”	-	• protocol of input data • temperature field
”name.eqa”	-	• field of equivalent age
”name.hyd”	-	• field of degree of hydratation

2.2 Example: Adiabatic System

At first a simple adiabatic system is considerd. In the next four tables the complete input and output data are documented.

A concrete cube of 1m^3 was considered. The concrete CO3 was chosen. The calculation time is 24h. From the output file the development of the heat source can be derived.

File 2-1: bench_01.dat

```
[name of program]
     Etahb

[file]
```

```
     bench_01      ; adiabatic environment

[geometry]
     0.4           ; in [m]    width (thickness) of the first substructure
     0.3           ; in [m]    width (thickness) of the second substructure
     0.3           ; in [m]    width (thickness) of the third substructure

[time]
     24            ; in [h] time of analysis

[increments]
     0.1           ; in [m]  local increment dz

[boundary conditions]
      0.0          ; in [°C]            temperature of environment at z=0
      0.0          ; in [W/(m*m K)]     heat transfer coefficient before removal at z=0
      0.0          ; in [W/(m*m K)]     heat transfer coefficient after removal at z=0
      0.0          ; in [h]             time of removal of formwork etc.

      0.0          ; in [°C]            temperature of enviroment at z=d
      0.0          ; in [W/(m*m K)]     heat transfer coeffizient before removal at z=d
      0.0          ; in [W/(m*m K)]     heat transfer coeffizient after removal at z=d
      0.0          ; in [h]             time of removal of formwork etc.
```

File 2-2: bench_01.mat

```
[name of program]
     Etahb

[file]
     bench_01              ; adiabatic environment

[material 1]
     1                     ; material number
     CONCRETE C03          ; name of material
     0.3000                ; spec. heat capacity   [W h / kg K]
     -1.080                ; heat source model parameter (JONASSON) exponent c_1 [-]
     2.1                   ; heat conductivity [W / m K]
     149044                ; max. heat realease of concrete [kJ / (m*m*m)]
     2336                  ; density [kg / m*m*m]
     12.0                  ; initial temperature at t=0 h
     26.8385               ; heat source model parameter (JONASSON) denominator t_k [h]
     280                   ; cement content [kg / m*m*m]

[material 2]
     2                     ; material number
     CONCRETE C03          ; name of material
     0.3000                ; spec. heat capacity   [W h / kg K]
     -1.080                ; heat source model parameter (JONASSON) exponent c_1 [-]
     2.1                   ; heat conductivity [W / m K]
     149044                ; max. heat realease of concrete [kJ / (m*m*m)]
     2336                  ; density [kg / m*m*m]
     12.0                  ; initial temperature at t=0 h
     26.8385               ; heat source model parameter (JONASSON) denominator t_k [h]
     280                   ; cement content [kg / m*m*m]

[material 3]
     3                     ; material number
     CONCRETE C03          ; name of material
     0.3000                ; spec. heat capacity   [W h / kg K]
     -1.080                ; heat source model parameter (JONASSON) exponent c_1 [-]
     2.1                   ; heat conductivity [W / m K]
     149044                ; max. heat realease of concrete [kJ / (m*m*m)]
     2336                  ; density [kg / m*m*m]
     12.0                  ; initial temperature at t=0 h
     26.8385               ; heat source model parameter (JONASSON) denominator t_k [h]
     280                   ; cement content [kg / m*m*m]
```

File 2-3: bench_01.tem

```
     [name of program]
     Etahb

     [files]
     output: bench_01.tem
     output: bench_01.eqa
```

```
output: bench_01.hyd
input: bench_01.dat
input: bench_01.mat

[material 1]
number:   1
name:     CONCRETE C03
spec. heat capacity c_c:  0.3000 [W h / (kg K)]
heat source modell parameter (JONASSON) exponent c_1: -1.0800 [-]
heat conductivity lamda:  2.1000 [W / (m K)]
max. heat release of concrete maxQ:  149044 [kJ / (m*m*m)]
desity rho: 2336.0 [kg / (m*m*m)]
initial temperatur T_0:   12.00 [°C]
heat source modell parameter (JONASSON) denominator t_k:  26.84 [h]
cement content C:  280.0 [kg / (m*m*m)]

[material 2]
number:   2
name:     CONCRETE C03
spec. heat capacity c_c:  0.3000 [W h / (kg K)]
heat source modell parameter (JONASSON) exponent c_1:  -1.0800 [-]
heat conductivity lamda:  2.1000 [W / (m K)]
max. heat release of concrete maxQ:  149044 [kJ / (m*m*m)]
density rho: 2336.0 [kg / (m*m*m)]
initial temperatur T_0:  12.00 [°C]
heat source modell parameter (JONASSON) denominator t_k:  26.84 [h]
cement content C:  280.0 [kg / (m*m*m)]

[material 3]
number:   3
name:     CONCRETE C03
spec. heat capacity c_c:  0.3000 [W h / (kg K)]
heat source modell parameter (JONASSON) exponent c_1:  -1.0800 [-]
heat conductivity lamda:  2.1000 [W / (m K)]
max. heat release of concrete maxQ:  149044 [kJ / (m*m*m)]
density rho: 2336.0 [kg / (m*m*m)]
initial temperatur T_0:  12.00 [°C]
heat source modell parameter (JONASSON) denominator t_k:  26.84 [h]
cement content C:  280.0 [kg / (m*m*m)]

[geometry]
substructure 1 width =    0.40 [m]
substructure 2 width =    0.30 [m]
substructure 3 width =    0.30 [m]

[boundary conditions]
z = 0
ambient temperature:  0.0 [°C]
heat transfer coefficient before removal:  0.0 [W / (m*m K)]
heat transfer coefficient after removal:  0.0 [W / (m*m K)]
time of removal:  0.0 [h]

z = d
ambient temperature:  0.0 [°C]
heat transfer coefficient before removal:  0.0 [W / (m*m K)]
heat transfer coefficient after removal:  0.0 [W / (m*m K)]
time of removal:  0.0 [h]


[system parameter]
thickness:    1.0 [m]
number of local increments:   10 [-]
local increment:   0.100 [m]
time:   24.0 [h]
number of time increments:    8640 [-]
time increment:   0.003 [h]

[temperature diffusivity]
material 1:  a=    0.003 [m*m/h]
material 2:  a=    0.003 [m*m/h]
material 3:  a=    0.003 [m*m/h]

[convergence criteria]
material 1: a_m1 * dt / (dz * dz) =   0.00077551
material 2: a_m2 * dt / (dz * dz) =   0.00077551
material 3: a_m3 * dt / (dz * dz) =   0.00077551
```

[temperature]

t \ z	0.050	0.150	0.250	0.350	0.450	0.550	0.650	0.750	0.850	0.950
1.00	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000
2.00	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000
3.00	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000
4.00	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000
5.00	12.001	12.001	12.001	12.001	12.001	12.001	12.001	12.001	12.001	12.001
6.00	12.005	12.005	12.005	12.005	12.005	12.005	12.005	12.005	12.005	12.005
7.00	12.018	12.018	12.018	12.018	12.018	12.018	12.018	12.018	12.018	12.018
8.00	12.050	12.050	12.050	12.050	12.050	12.050	12.050	12.050	12.050	12.050
9.00	12.108	12.108	12.108	12.108	12.108	12.108	12.108	12.108	12.108	12.108
10.00	12.201	12.201	12.201	12.201	12.201	12.201	12.201	12.201	12.201	12.201
11.00	12.334	12.334	12.334	12.334	12.334	12.334	12.334	12.334	12.334	12.334
12.00	12.511	12.511	12.511	12.511	12.511	12.511	12.511	12.511	12.511	12.511
13.00	12.733	12.733	12.733	12.733	12.733	12.733	12.733	12.733	12.733	12.733
14.00	13.000	13.000	13.000	13.000	13.000	13.000	13.000	13.000	13.000	13.000
15.00	13.312	13.312	13.312	13.312	13.312	13.312	13.312	13.312	13.312	13.312
16.00	13.669	13.669	13.669	13.669	13.669	13.669	13.669	13.669	13.669	13.669
17.00	14.067	14.067	14.067	14.067	14.067	14.067	14.067	14.067	14.067	14.067
18.00	14.506	14.506	14.506	14.506	14.506	14.506	14.506	14.506	14.506	14.506
19.00	14.983	14.983	14.983	14.983	14.983	14.983	14.983	14.983	14.983	14.983
20.00	15.496	15.496	15.496	15.496	15.496	15.496	15.496	15.496	15.496	15.496
21.00	16.041	16.041	16.041	16.041	16.041	16.041	16.041	16.041	16.041	16.041
22.00	16.616	16.616	16.616	16.616	16.616	16.616	16.616	16.616	16.616	16.616
23.00	17.218	17.218	17.218	17.218	17.218	17.218	17.218	17.218	17.218	17.218
24.00	17.843	17.843	17.843	17.843	17.843	17.843	17.843	17.843	17.843	17.843

date:
23.08.1998
running time:
0 : 0 : 3 : 57

File 2-4: bench_01.eqa

[equivalent age]

t \ z	0.050	0.150	0.250	0.350	0.450	0.550	0.650	0.750	0.850	0.950
1.00	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
2.00	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20
3.00	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
4.00	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.40
5.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
6.00	3.60	3.60	3.60	3.60	3.60	3.60	3.60	3.60	3.60	3.60
7.00	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20	4.20
8.00	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80	4.80
9.00	5.40	5.40	5.40	5.40	5.40	5.40	5.40	5.40	5.40	5.40
10.00	6.01	6.01	6.01	6.01	6.01	6.01	6.01	6.01	6.01	6.01
11.00	6.62	6.62	6.62	6.62	6.62	6.62	6.62	6.62	6.62	6.62
12.00	7.24	7.24	7.24	7.24	7.24	7.24	7.24	7.24	7.24	7.24
13.00	7.87	7.87	7.87	7.87	7.87	7.87	7.87	7.87	7.87	7.87
14.00	8.52	8.52	8.52	8.52	8.52	8.52	8.52	8.52	8.52	8.52
15.00	9.18	9.18	9.18	9.18	9.18	9.18	9.18	9.18	9.18	9.18
16.00	9.85	9.85	9.85	9.85	9.85	9.85	9.85	9.85	9.85	9.85
17.00	10.55	10.55	10.55	10.55	10.55	10.55	10.55	10.55	10.55	10.55
18.00	11.26	11.26	11.26	11.26	11.26	11.26	11.26	11.26	11.26	11.26
19.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
20.00	12.77	12.77	12.77	12.77	12.77	12.77	12.77	12.77	12.77	12.77
21.00	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56
22.00	14.39	14.39	14.39	14.39	14.39	14.39	14.39	14.39	14.39	14.39
23.00	15.24	15.24	15.24	15.24	15.24	15.24	15.24	15.24	15.24	15.24
24.00	16.13	16.13	16.13	16.13	16.13	16.13	16.13	16.13	16.13	16.13

File 2-5: bench_01.hyd

[degree of hydratation]

t \ z	0.050	0.150	0.250	0.350	0.450	0.550	0.650	0.750	0.850	0.950
1.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

8.00	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
9.00	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
10.00	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
11.00	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
12.00	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
13.00	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013
14.00	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018
15.00	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024
16.00	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
17.00	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037
18.00	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045
19.00	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053
20.00	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063
21.00	0.072	0.072	0.072	0.072	0.072	0.072	0.072	0.072	0.072	0.072
22.00	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083
23.00	0.093	0.093	0.093	0.093	0.093	0.093	0.093	0.093	0.093	0.093
24.00	0.105	0.105	0.105	0.105	0.105	0.105	0.105	0.105	0.105	0.105

2.3 Example: Slab on Ground System

It is often necessary to study slab on ground. The next two tables show the complete input data. Assume a slab with a thickness of 1.5*m*.

To simulate a infinitely thick ground (sand/gravel), with a thickness below the slab of 9.0*m*. The corresponding heat transfer coefficients are chosen to 10000 W/m^2K. The ambient temperature for ground is chosen to initial temperature of ground (*at z=d:* $T_a = T_{S0}$). This two conditions ensure that temperature at *z=d* remains at ambient temperature.

At 72*h* the formwork was removed.

File 2-6: bench_02.dat

```
[name of program]
     Etahb

[file]
     bench_02       ; slab on ground system

[geometry]
     1.5            ; in [m]    width (thickness) of the first substructure
     4.5            ; in [m]    width (thickness) of the second substructure
     4.5            ; in [m]    width (thickness) of the third substructure

[time]
     2016           ; in [h]    time of analysis

[increments]
     0.15           ; in [m]    local increment dz

[boundary conditions]
     10.0           ; in [°C]           temperature of environment at z=0
      4.0           ; in [W/(m*m K)]    heat transfer coefficient before removal at z=0
     30.0           ; in [W/(m*m K)]    heat transfer coefficient after removal at z=0
     72.0           ; in [h]            time of removal

     12.0           ; in [°C]           temperature of environment at z=d
  10000.0           ; in [W/(m*m K)]    heat transfer coefficient before removal at z=d
  10000.0           ; in [W/(m*m K)]    heat transfer coefficient after removal at z=d
     10.0           ; in [h]            time of removal
```

File 2-7: bench_02.mat

```
[name of program]
     Etahb

[file]
     bench_02                ; slab on ground system

[material 1]
     1                       ; material number
     concrete CO3            ; name of material
     0.3000                  ; spec. heat capacity   [W h / kg K]
     -1.080                  ; heat source model parameter (JONASSON) exponent c_1 [-]
     2.1                     ; heat conductivity [W / m K]
     149044                  ; max. heat realease of concrete maxQ: [kJ/(m*m*m)]
     2336                    ; density [kg / m*m*m]
     12.0                    ; initial temperature at t=0 h
     26.8385                 ; heat source model parameter (JONASSON) denominator t_k [h]
     280                     ; cement content [kg / m*m*m]

[material 2]
     2                       ; material number
     sandy ground            ; name of material
     0.23693                 ; spec. heat capacity   [W h / kg K]
     1.0                     ; heat source model parameter (JONASSON) exponent c_1 [-]
     1.0                     ; heat conductivity [W / m K]
     0.0                     ; max. heat realease of concrete [kJ / (m*m*m)]
     1700                    ; density [kg / m*m*m]
     12.0                    ; initial temperature at t=0 h
     1.0                     ; heat source model parameter (JONASSON) denominator t_k [h]
     0.0                     ; cement content [kg / m*m*m]

[material 3]
     3                       ; material number
     sandy ground            ; name of material
     0.23693                 ; spec. heat capacity   [W h / kg K]
     1.0                     ; heat source model parameter (JONASSON) exponent c_1 [-]
     1.0                     ; heat conductivity [W / m K]
     0.0                     ; max. heat realease of concrete [kJ / (m*m*m)]
     1700                    ; density [kg / m*m*m]
     12.0                    ; initial temperature at t=0 h
     1.0                     ; heat source model parameter (JONASSON) denominator t_k [h]
     0.0                     ; cement content [kg / m*m*m]
```

3 ANNEX

For the computation, a set of material values, parameters of the used material models, initial and boundary conditions are presented.

Table 3-1: concrete composition

CONCRETE COMPOSITION		CO1	CO2	CO3
class	*[MPa]*	25	35	25
cement type	*[-]*	CEM I 32.5R	CEM III/B 32.5 NWHSNA	CEM I 32.5R
cement content	*[kg/m^3]*	270	390	280
fly ash	*[kg/m^3]*	60	-	220
water-cement ratio *(W/(C+0.3 FA))*	*[-]*	0.61	0.47	0.57

aggregate	[kg/m³]	1848	1765	1852

Table 3-2: material values

PARAMETER			CO1	CO2	CO3	GROUND
heat conductivity	λ_c	[W/mK]	2.1	2.6	2.3	1.0
heat capacity	c_c	[Wh/kgK]	0.300	0.300	0.300	0.23693
density	ρ_c	[kg/m³]	2350	2390	2336	1700
heat of hydratation of cement	maxQ	[kJ/ m³]	138396	144296	149044	-

Table 3-3: parameter of material models

MATERIAL MODEL	PARAMETER		CO1	CO2	CO3	GROUND	REMARKS
heat source (Jonasson)	t_k	[h]	12.0548	16.19731	26.8385	1.0	
	c_1	[-]	-1.13489	-1.18937	-1.08	1.0	
compressive strenght (IBMB)	f_{c1}	[MPa]	47.89	57.123	55.21	-	not used in temperature calculation
	α_0	[-]	0.1995	0.3592	0.2462	-	
tensile strenght (IBMB)	f_{ct1}	[MPa]	3.005	3.368	3.31	-	
	α_0	[-]	0.1995	0.3592	0.2462	-	
Young's modulus (IBMB)	E_{ct1}	[GPa]	36.630	35717	36386	-	
	α_0	[-]	0.1995	0.3592	0.2462	-	

Table 3-4: initial and boundary conditions

WEATHER / SEASON			SPRING / AUTUMN	SUMMER	WINTER
fresh concrete temperature	T_{c0}	[°C]	15	25	10
inital temperature of ground	T_{s0}	[°C]	8	12	5
ambient temperature	T_a	[°C]	12	25	7

coeff. of heat transfer immediatley after casting with curing protective layer	α_e	*[W/m²K]*	10	10	10
coeff. of heat transfer immediatley after removal of protective layer	α_e	*[W/m²K]*	20	20	30

Table 3-5: terms and definitions

TERM	EXPLANATION	UNIT
ρ	density	*[kg/m³]*
α_0	initial degree oh hydration	*[-]*
λ_c	heat conductivity	*[W/mK]*
α_e	heat transfer coefficient	*[W/m²K]*
a	temperature diffusitivity, $a = \lambda / (\rho\ c)$	*[m²/h]*
c_c	heat capacity	*[Wh/kgK]*
c_1	parameter of JONASSON heat source model, exponent	*[-]*
dt	constant time increment	*[h]*
dz	constant local increment	*[m]*
E_{ct1}	end value of Young's modulus material model, theor. value at $\alpha=1$	*[GPa]*
f_{c1}	end value of compressive strength material model, theor. value at $\alpha=1$	*[MPa]*
f_{ct1}	end value of tensile strength material model, theor. value at $\alpha=1$	*[MPa]*
$maxQ$	instrinsic heat of hydratation of concrete	*[kJ/m³]*
H_C	instrinsic heat of hydratation of cement	*[kJ/kg]*
m	number of time increments	*[-]*
n	number of local increments	*[-]*
T_a	ambient temperature	*[°C]*
T_{c0}	initial temperature of concrete	*[°C]*
t_i	actual time (age)	*[h]*
t_k	parameter of JONASSON heat source model, denominator	*[h]*
T_{s0}	initial temperature of soil	*[°C]*

z_k	actual point	*[m]*

3.1 Important Remarks

Etahb was developed to study –on a realistic scale- the fields of temperatures and degree of hydratation in a massiv slab on ground. Such fields are pre-requisite for the computation of thermal stresses in young concrete. In order to successfully work with this engeneering tool, the following items must be taken into account:

- maximum numbers of local supporting points: *2000*

  thickness of strip $dz \leq d / 2000$;

  z is the coordinate of center of strip from left hand side.

- maximum numbers of points of time integration t_i: *368800*; time increment $dt=1/360\ h$.

- the convergence of the explicit differences scheme requires the following stability condition:

$$F_\sigma = \lambda / (\rho\ c)\ dt / dz^2 \leq 0.5.$$

  This condition is necessary, but not adequate. To check the convergence the user have to double the local interval and calculate the bench again. If the difference between this two results is small, the stability condition is fullfilled. If $F_\sigma > 0.5$ at the beginning of the calculation, the user has to interrupt the analysis and double the local interval again. Otherwise the numerical results may be wrong and oscilate.

- practical values for dz are *5cm - 30cm.*

- a calculation of a slab on ground system with *dz=0.01cm, d=10.5m, t=2084h* is runing about 15min on a AMD K6 PC.

- the program does not check errors of the user in input data. Error messages of the system could easily corrected with the protocol in the "NAME.TEM"- file.

APPENDIX C

PROBLEMS AND HOMEWORK

PROBLEM 1

For the standard concretes of App. A the chemical composition of the PC-clinkers, the mass precentage of PC-clinker in the cement and the fly ash contents are given in App. A.

- Determine the heat releases of the cements, including slag and fly ash. Calculate the maximum heat release max Q and the maximum adiabatic temperature rise max ΔT_{ad} (sec. 2 and 4, App. A).
- In which seasons would you apply these concrete mixes and for which structures? How is your recommendation affected by the thickness of e.g. a foundation slab?
- Do not hesitate to propose alternatives with subsequent justification (also with respect to cost).

PROBLEM 2 DEGREE OF HYDRATION

Heat release, heating up of concrete member, evolution of properties and stresses etc. are influenced by the degree of hydration.

- Determine $\alpha(t_e)$ with the parameters t_k and c_1 for the standard concretes CO1 to CO3 with the data of App. A.
- How would you estimate max ΔT_{ad} and $\alpha(t_e)$ in the pre-planning phase (no test results available!)?
- For the standard concretes $\alpha(t_e)$; t_k and c_1 are known. Model alternatively $\alpha(t_e)$ on basis of the ISCE-standard.

PROBLEM 3

- Compute with program ETAHB the fields of concrete temperature $T(z,t)$ and of degree of hydration $\alpha(z,t)$ using the properties of concretes CO1 to CO3 for several values of slab thickness.
- Plot $T(z,t)$ and $\alpha(z,t)$ versus time (age) up to 1000 s.
- Discuss the results, compare them. Make proposals as to modfications to lower the temperature rise. Justify your proposals.
- Compute the average values of tensile strength and modulus of elasticity dependent on time for CO1 to CO3 using the given α_0-values and parameters over 1000 hrs.

PROBLEM 4

- Because you know the field $T(z,t)$, you are able to determine the free curvature $æ_0(t)$ and the mean free thermal strain $\varepsilon_{0m}(t)$ for the slabs, e.g. for one of the standard concretes, s. sect. 6 and 7 of App. A.
- Determine the restraint stress in outer fibers (strips) in top and bottom region of slab of chosen thickness d and length L.
- Will cracking occur?
- Present methods to avoid thermal cracking?